Le système SI et son évolution; le rôle de la seconde dans le future SI

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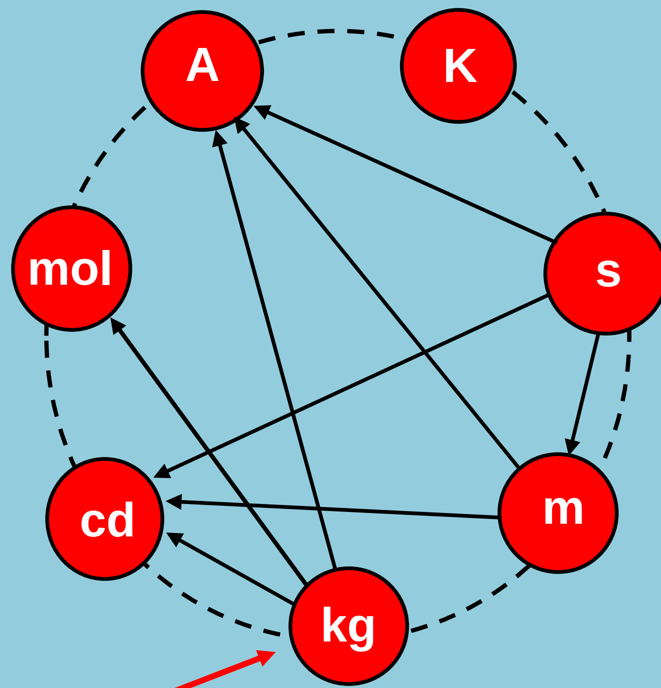
TJQ and Jean Kovalevsky at the French Archives on 16 November 2010 with the Metre and Kilogram of the Archives deposited there on 22 June 1799

Photo TJQ

The safe in the vault of the prototypes at the BIPM where the kilogram and metre rested from 1889 until 1998 when they were transferred to a new safe.



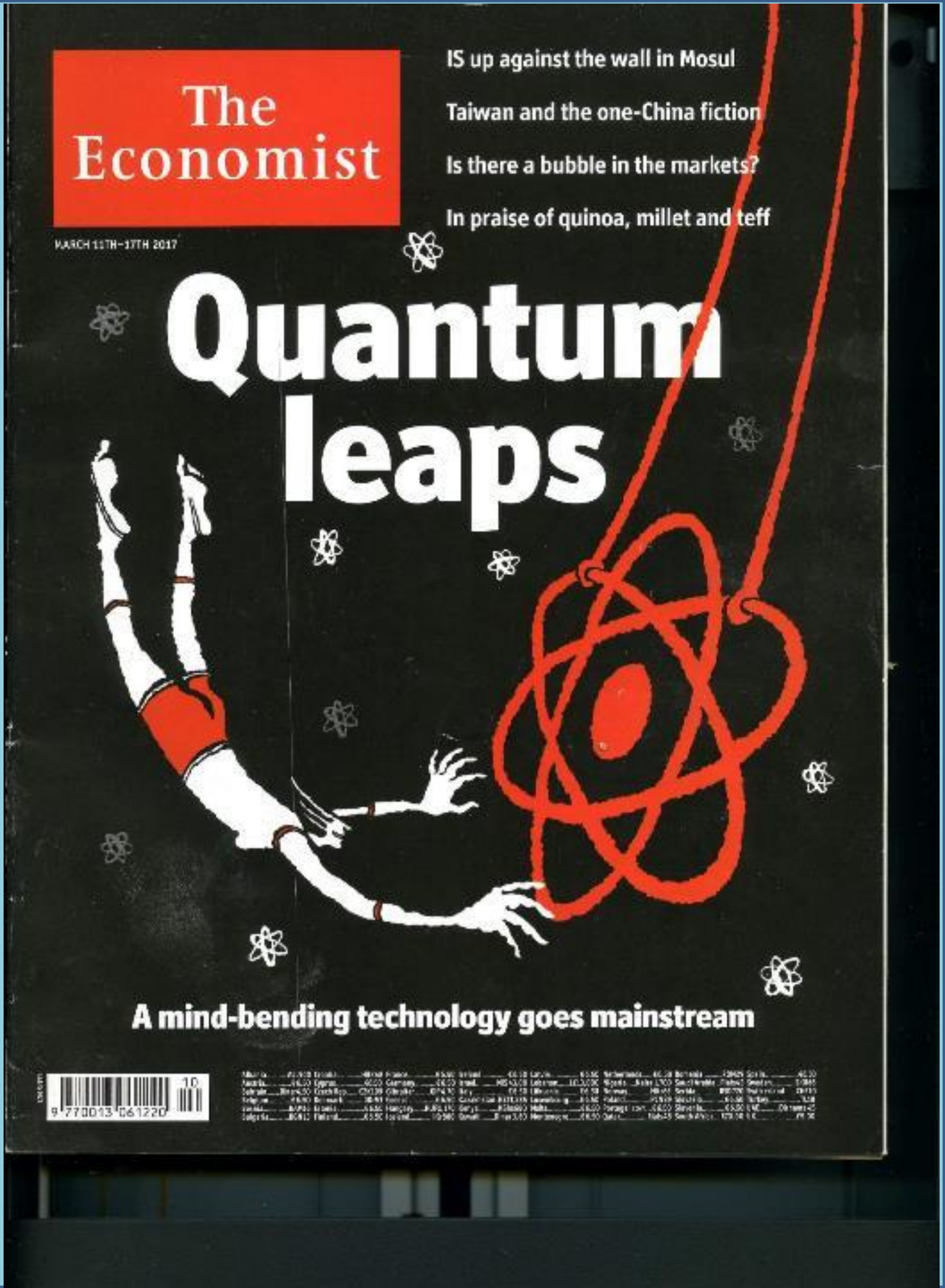
The base units of the SI

situation today

The SI

In November 2018 the SI embraces quantum physics.

From artefacts to atoms at last!



26^{ieme} CGPM novembre 2018:

le Système international d'unités, le SI, sera le système d'unités selon lequel :

- la fréquence de la transition hyperfine dans l'état fondamental de l'atome de césium 133 $\Delta\nu Cs$ est 9 192 631 770 hertz,

- la vitesse de la lumière dans le vide c est 299 792 458 mètres par seconde,

- la constante de Planck h est 6.626 070 xx $\times 10^{-34}$ joule seconde,

- la charge élémentaire e est 1.602 176 6xx $\times 10^{-19}$ coulomb,

- la constante de Boltzmann k est égale à 1.380 648 xx $\times 10^{-23}$ joule par kelvin,

- la constante d'Avogadro N_A est 6.022 140 xx $\times 10^{23}$ par mole,

- l'efficacité lumineuse K_{cd} d'un rayonnement monochromatique de fréquence 540 $\times 10^{12}$ Hz est 683 lumens par watt,

Table 1. The seven defining constants of the SI, and the seven corresponding units they define

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Defining constant	Symbol	Numerical value	Unit
hypererfine transition frequency of Cs	$\Delta\, v\, Cs$	9 192 631 770	Hz (**s**$^{-1}$)
speed of light in vacuum	c	299 792 458	**m** s^{-1}
Planck constant	h	6.626 070 xx ×10−34	**kg** m^{2} s^{-1}
elementary charge	e	1.602 176 6xx ×10−19	C (**A**s)
Boltzmann constant	k	1.380 648 xx ×10−23	kg m^{2} s^{-2} **K**$^{-1}$
Avogadro constant	N_A	6.022 140 xx ×1023	**mol**$^{-1}$
luminous efficacy	K_{cd}	683	**cd** sr kg^{-1} m^{-2} s^{3} (lm W^{-1})

This shows the hierarchical nature of the new definitions. We begin with the second, then knowing the second we can define the metre, then knowing the second and the metre we can define the kilogram and so on…The mole is the only one that does not include any of the other units.

What does it mean to define the numerical value of a fundamental constant of physics – surely these are fixed by nature?

The value of a constant of physics is indeed fixed by nature but its numerical value depends on the size of the unit with which we choose to measure it, take for example the speed of light:

the speed of light, c may be written:

$$c = 299\,792\,458 \text{ metres per second}$$

or $$c = 983\,571\,056.4 \text{ feet per second}$$

or $$c = 327\,857\,018.8 \text{ yards per second}$$

$$\text{the value of } c = \text{numerical value} \times \text{unit}$$

The value of c is a constant of nature.

1. If we define the units independently, then we must determine the numerical value of c by experiment, and it will have an uncertainty. That was the situation before 1983, when both the metre and the second were independently defined.

2. If the second is independently defined in terms of the frequency of the caesium transition, and we choose to fix the numerical value of c, then the effect is to define the size of the unit, equal to 299 792 458 in the case of the metre. This is the current definition of the metre, since the change in 1983. The numerical value now has zero uncertainty.
3. We have thus defined the metre in terms of a fixed numerical value for the speed of light.

How do we make practical use of such a definition?

We need to find an equation of physics that links the speed of light to length without including any unknown constants or quantities that themselves depend on length, such an equation is

$$c = \lambda f$$

Where λ is the wavelength of a light of frequency f.

We could also of course simply measure the time taken for a light signal to travel from one place to another but this is not practical for short distances.

Let us now look at the Planck constant, h

$$h \quad = \quad 6.626\,070xx \times 10^{-34} \quad \text{kg m}^2\ \text{s}^{-1}$$

$$\text{value of } h \quad = \quad \text{numerical value} \quad \times \quad \text{unit}$$

The value of h is a constant of nature.

1. If we define the unit kg m^2 s^{-1} independently, then we must determine the numerical value of h by experiment, and it will have an uncertainty. That is the present situation, this is what we are doing at present with watt (Kibble) balances and silicon spheres.

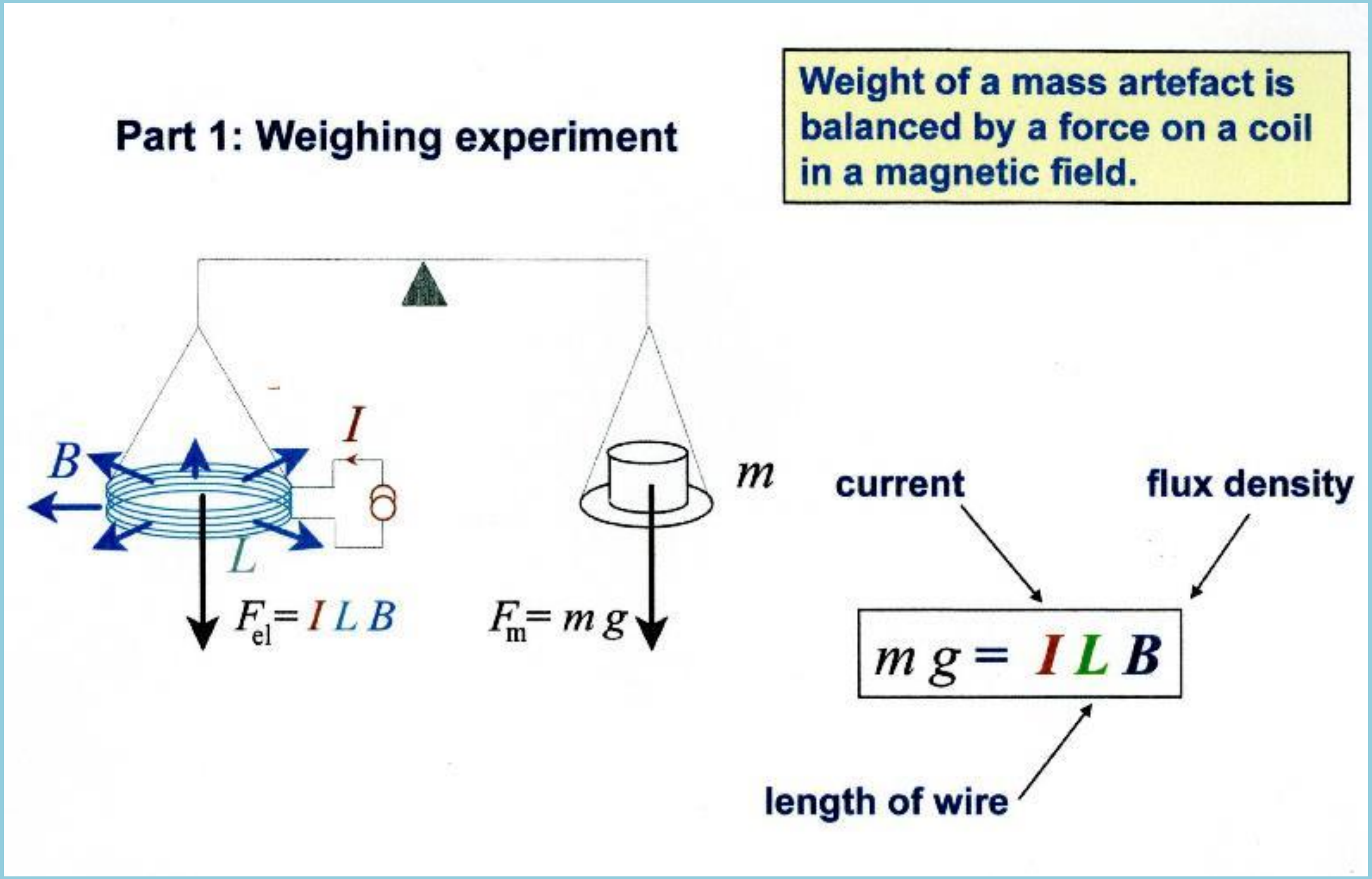
2. However, if the metre and the second are already independently defined, we can choose to fix the numerical value of h, then the effect is to define the size of the kilogram. This is the proposed new definition of the kilogram. The numerical value will have zero uncertainty. We just have to make sure we choose the right value i.e., one that is really consistent with the present definition of the kilogram. This is why we do it in two independent ways.

The key question is however the following: How can we be sure that the numerical value we choose for these defining constants are the right ones, i.e., that when the new definitions are implemented there will not be a step change in the size of the units?

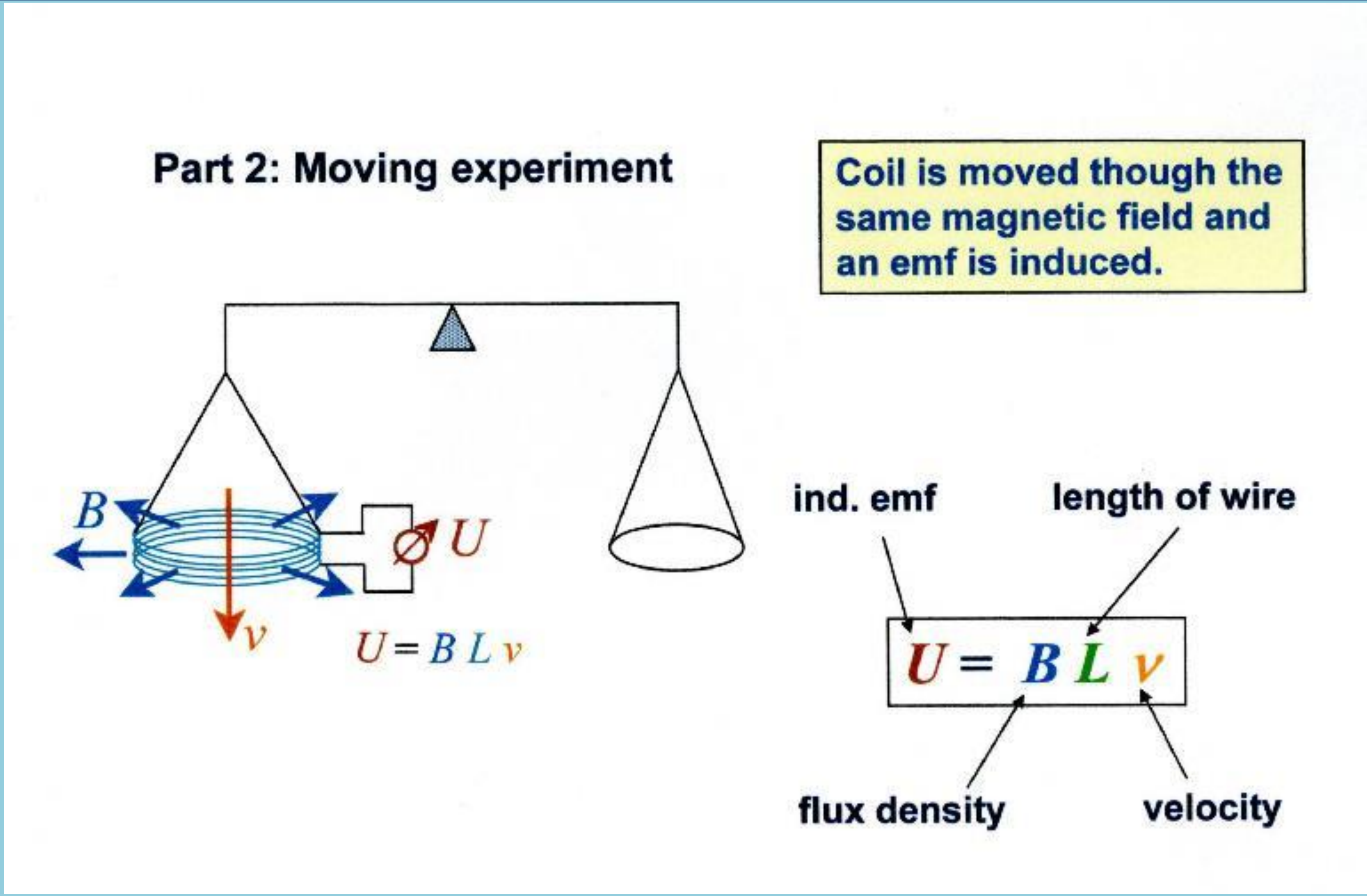
We do the measurements of the Planck constant by two methods:

The first is by means of a watt balance, which we now call the **Kibble balance** in which electrical power is compared with mechanical power to give a value for ***h***

The Kibble balance



But we cannot directly measure either *L* or *B* with sufficient accuracy

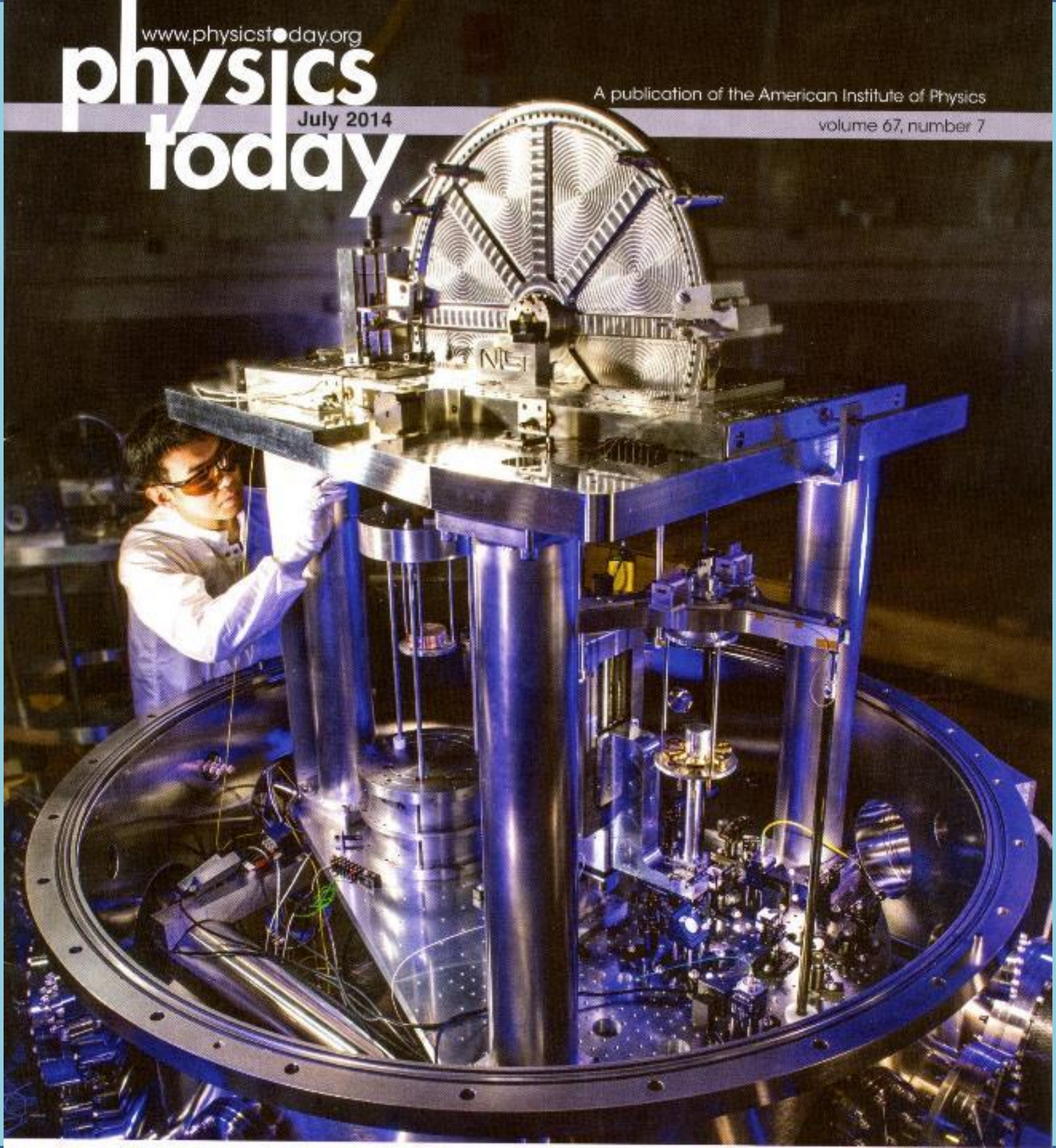


Combining the equations from the two configurations gives ***mgv = IU***
Taking advantage of the Josephson effect, which gives a voltage $U = nfh/2e$ and the quantum-Hall effect, which gives an electrical resistance $R = h/ie^2$ we can write

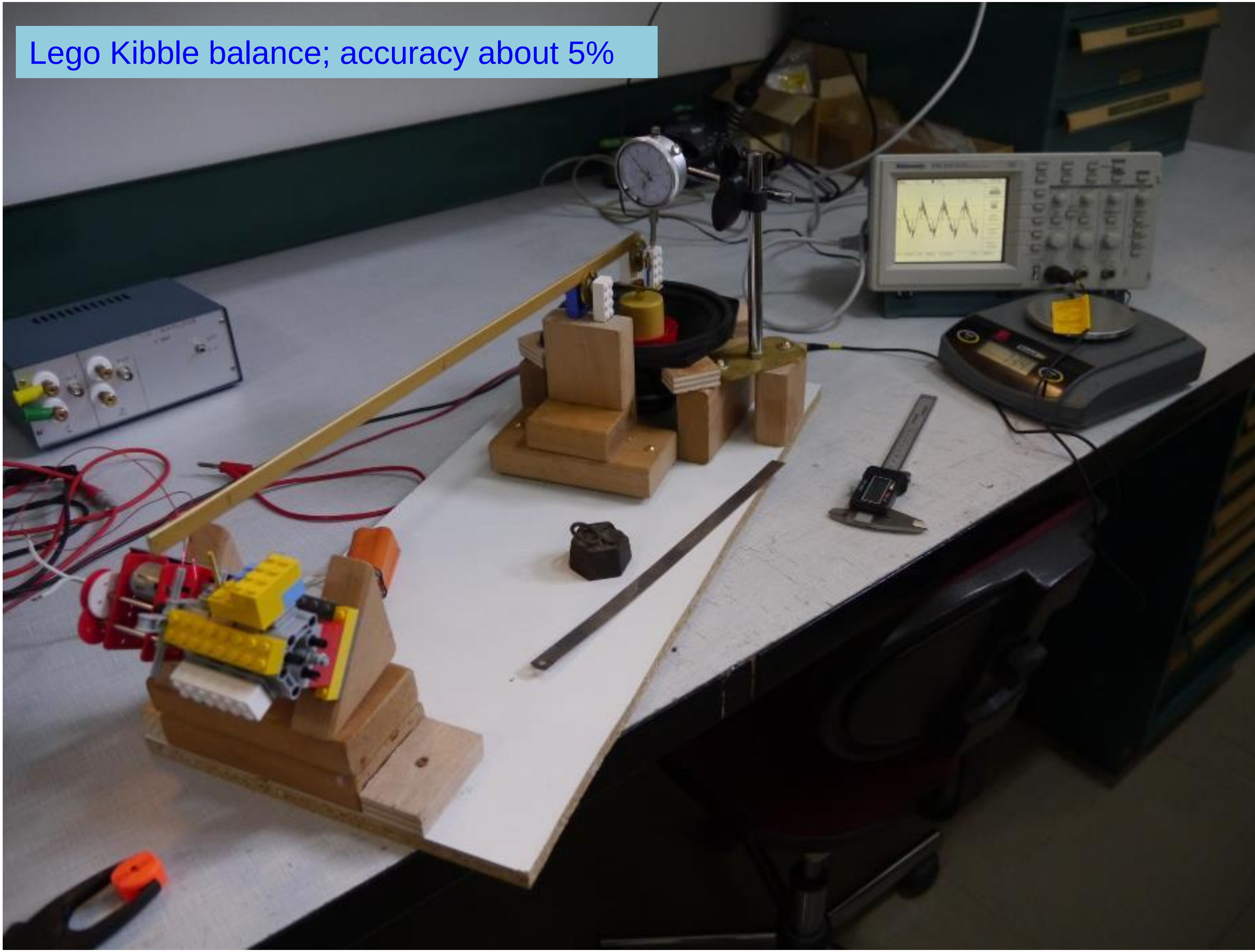
$IU = in^2f^2h/4$ so that $mgv = in^2f^2h/4$ which gives:

$$h = 4mgv/in^2f^2$$

NIST watt balance Mk II



Lego Kibble balance; accuracy about 5%



silicon spheres weighing about 1 kg containing about 215 253 842 $\times 10^{17}$ atoms

Photo BIPM

For the silicon crystal density we have:

$$\boldsymbol{N}_{\mathbf{A}} = n\, M(\mathrm{Si})/\rho\, a^3$$

Where n is the number of atoms per unit cell of silicon, M(Si) the molar mass of silicon, ρ the density of the sample of silicon and a its lattice constant so that, remembering that

$$\boldsymbol{N}_{\mathbf{A}}\boldsymbol{h} = [\boldsymbol{c\alpha}^2/2\boldsymbol{R}_\infty][\boldsymbol{M}_{\mathbf{u}}\, \boldsymbol{A}_{\mathbf{r}}(\mathbf{e})],$$

where α, R_∞, M_u and A_r(e) are the fine structure constant (known to parts in 10^{10}), the Rydberg for infinite mass (parts in 10^{12}), the molar mass constant (exact) and the relative atomic mass of the electron (parts in 10^{10}) respectively.

$$\boldsymbol{h}_{(\mathrm{silicon})} = [\boldsymbol{c\alpha}^2/2\boldsymbol{R}_\infty][\boldsymbol{M}_{\mathbf{u}}\, \boldsymbol{A}_{\mathbf{r}}(\mathbf{e})]\, \boldsymbol{\rho}\, \boldsymbol{a}^3 / \boldsymbol{n}\, \boldsymbol{M}(\mathbf{Si})$$

From the Kibble balance we have:

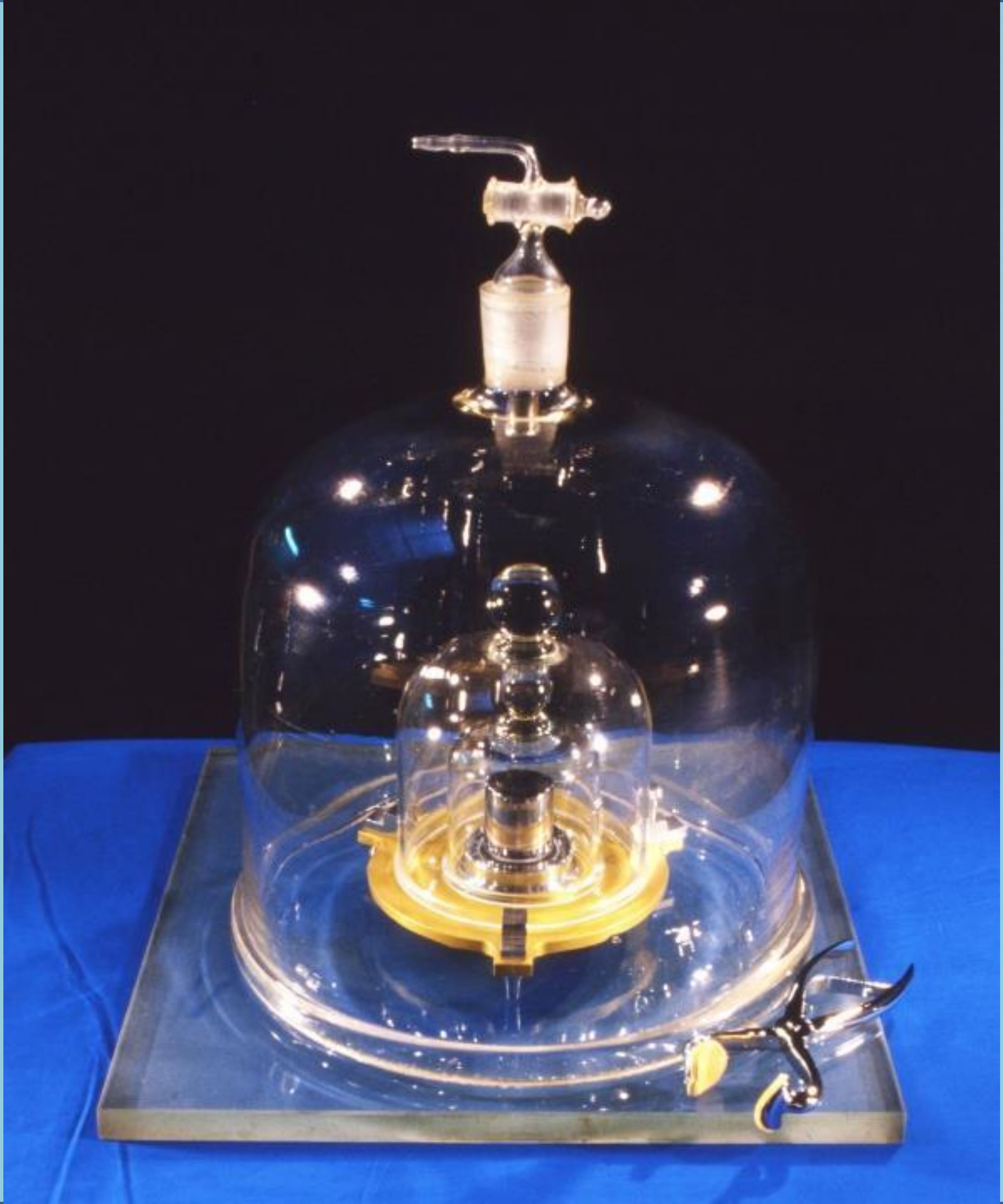
$$\boldsymbol{h = 4mgv/in^2f^2}$$

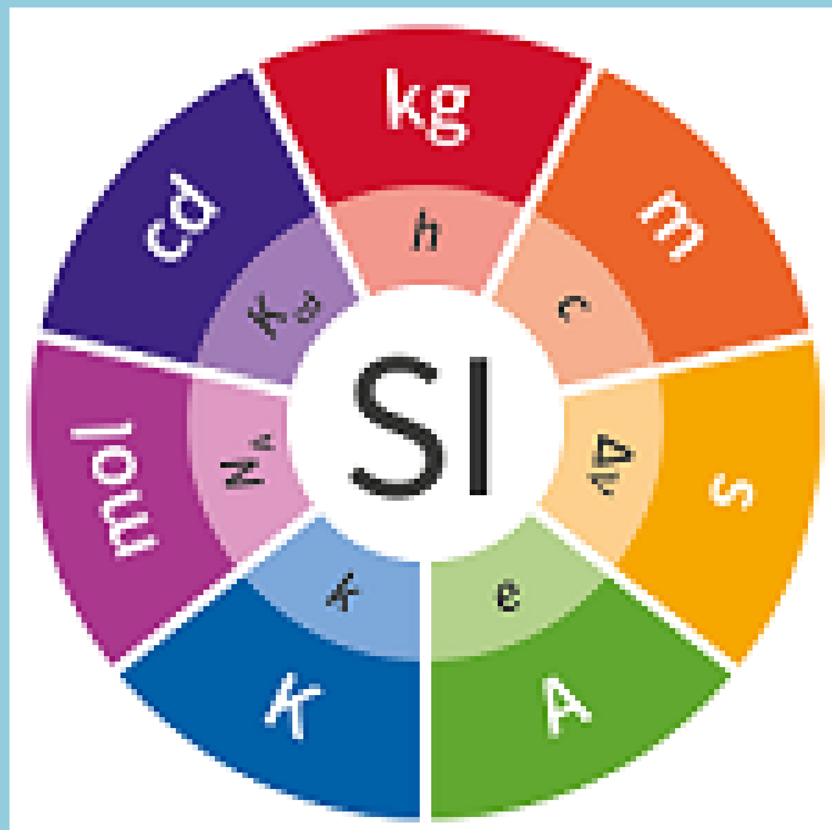
From the silicon sphere we have:

$$\boldsymbol{h_{(\mathrm{silicon})} = [c\alpha^2/2R_\infty][M_\mathrm{u}\, A_\mathrm{r}(\mathrm{e})]\, \rho\, a^3 / n\, M(\mathrm{Si})}$$

The two give the same value for *h* to within a few parts in 10^8 which is absolutely remarkable!

New BIPM set of reference mass standards. In the future their mass will be determined by the Kibble balance and silicon density determinations




SI
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