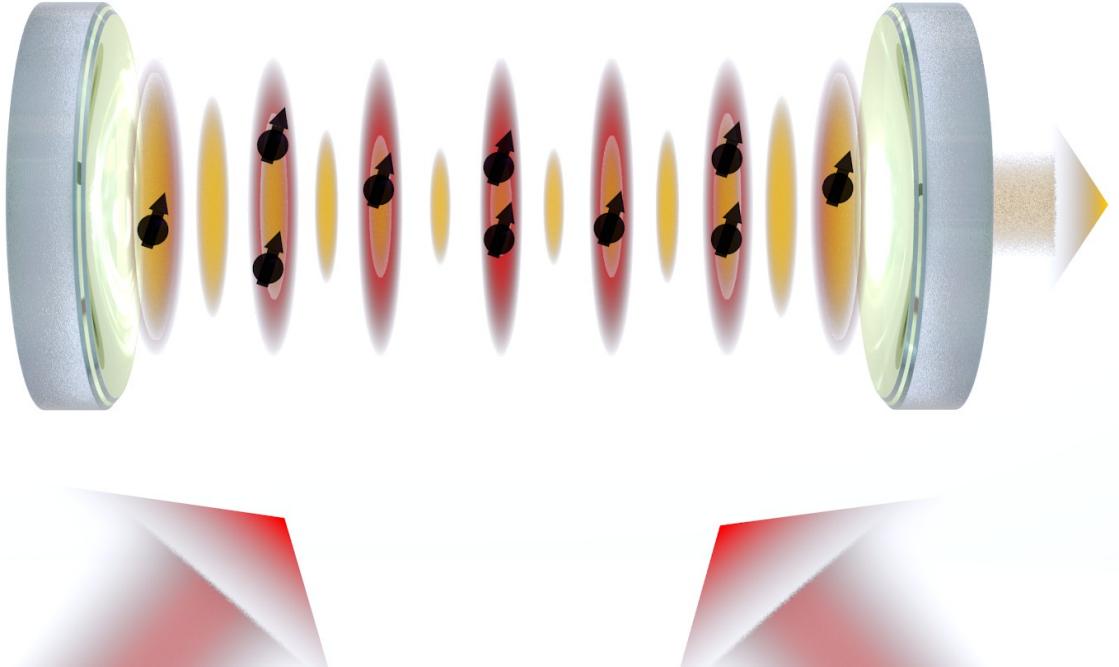


# SURECO : SUPERRADIANCE EN REGIME CONTINU

*Marion Delehaye*

*Martin Robert de Saint Vincent*



*Bruno Bellomo*  
*Martina Matusko (2020 - 2024)*  
*Martin Hauden (2021 - 2024)*  
*Jana El Badawi (2022 - 2025)*  
*Francisco Sebastian Ponciano-Ojeda (2021 – 2024)*  
*Joshua Ruelle (2024 - 2027)*

*Bruno Laburthe-Tolra*  
*Benjamin Pasquiou*  
*Gregoire Coget (2021)*  
*Ziyad Amodjee (2022)*  
*Yannis Pargoire (2024 - 2027)*

# CONTENTS



1 Introduction: motivations and principles

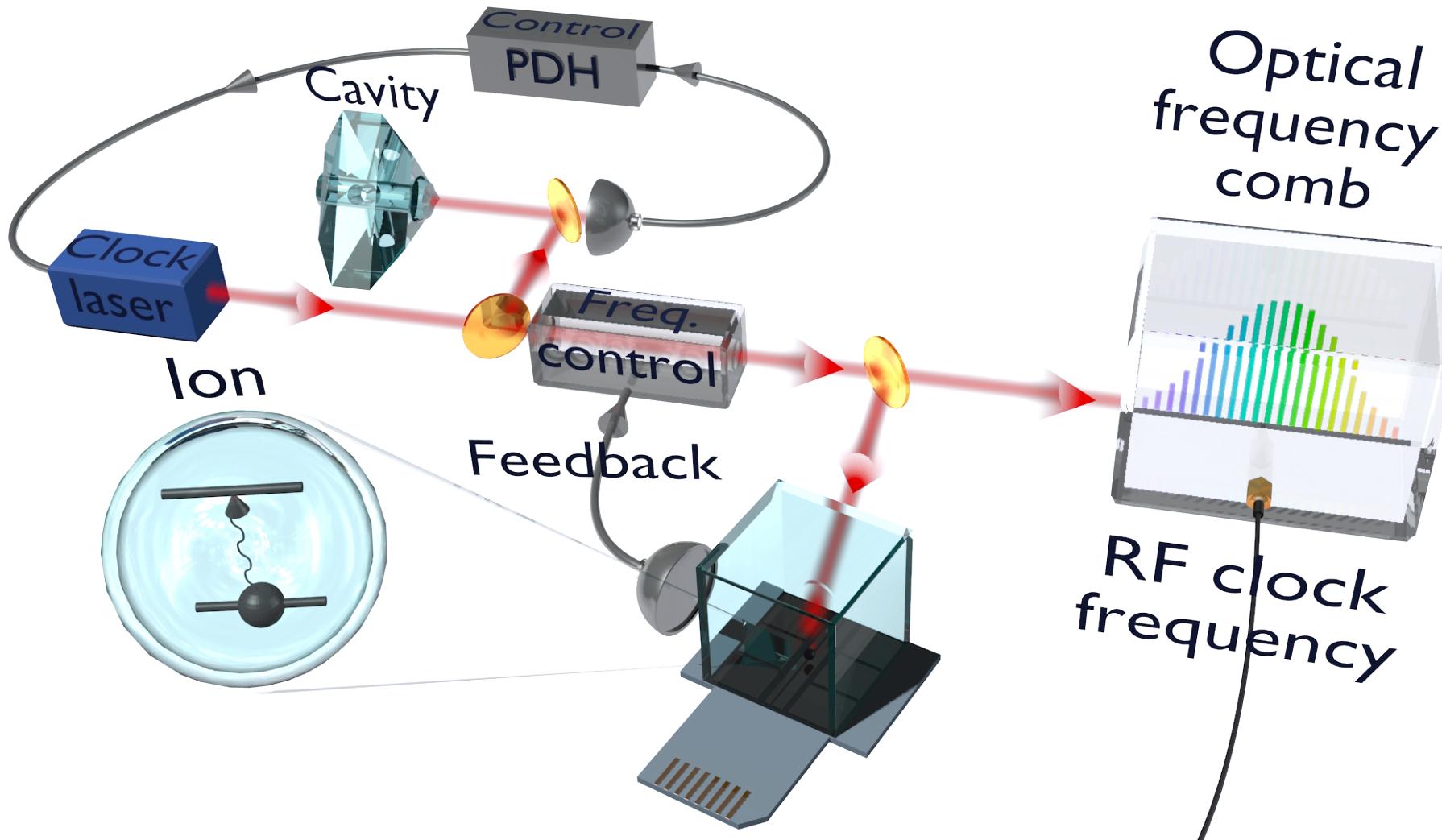
2 Challenge of the continuous operation

3 FEMTO-ST machine

4 LPL machine

5 Conclusion

# Passive optical clocks



Accuracy  $\sim 10^{-18}$

Stability  $\sim 10^{-16}$  @ 1s

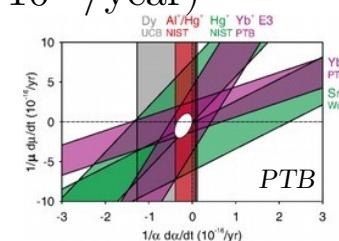
Bloom et al, Nature 506, 71 (2014)

# Need of better frequency references



- sensitive to potential variations of fundamental constants ( $\lesssim 10^{-16}/\text{year}$ )

Huntemann et al, PRL 113, 210802 (2014)



- sensitive to heights → relativistic geodesy

Lisdat et al, Nat. Comm 7, 12443 (2016)

gravitational redshift :  $10^{-16}/\text{m} \Rightarrow 10^{-18}$  : measurement of the geoid within a cm

→ monitor geophysical/plate subduction processes

Bondarescu et al, Geophys. J. Int. 202, 1770 (2015)

Tanaka&Katori, J. Geod. 95, 93 (2021)



- probably sensitive to dark matter ( $\sim 10^{-19}$ )

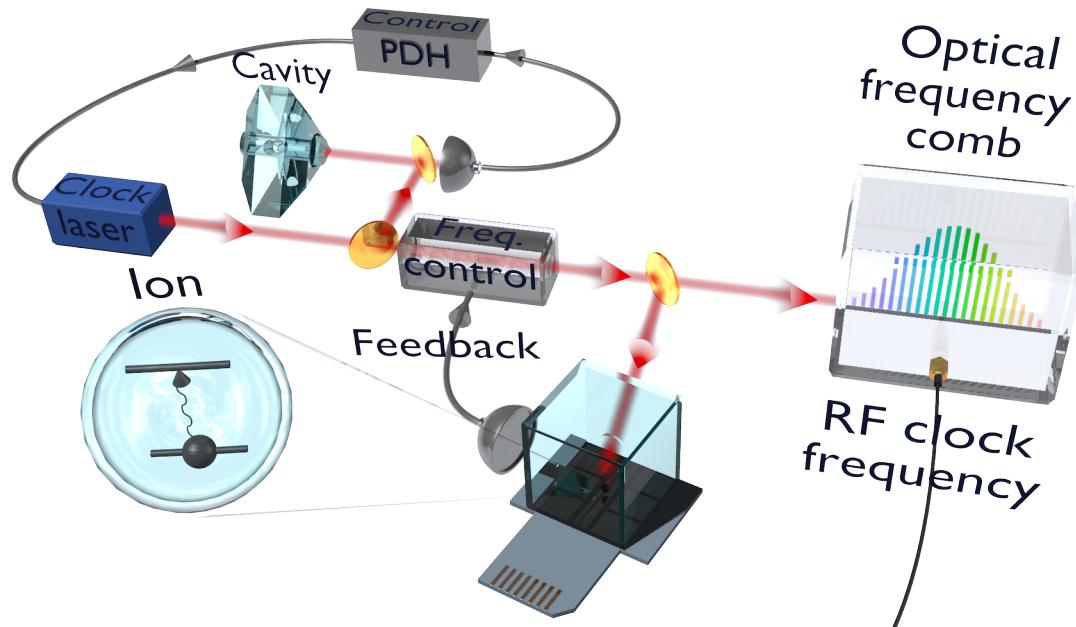
Derevianko, J. Phys.: Conf. Ser. 723, 012043

- (soon) sensitive to gravitational waves ( $\sim 10^{-20} \tau^{-1/2}$ ), on a different bandwidth than current detectors

Kolkowitz et al, PRD 94, 124043 (2016)



# Passive optical clocks: limitations



Accuracy  $\sim 10^{-18}$

Stability  $\sim 10^{-16}$  @ 1s

Bloom et al, Nature 506, 71 (2014)

Quantum Projection  
Noise (QPN)

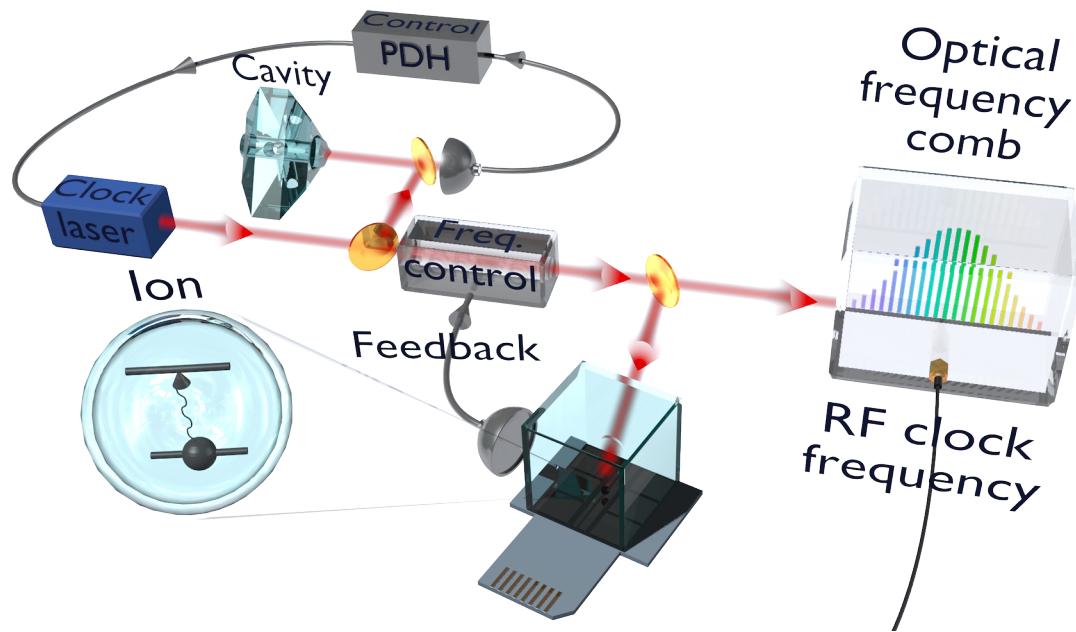
$$\sigma_{\text{QPN}}(\tau) = \frac{0.264}{\nu_c T_p} \sqrt{\frac{T_c}{N\tau}}$$

(Rabi interrogation)

$$\sigma_{\text{QPN}} \sim 10^{-17} \tau^{-1/2} \quad (N \sim 5000)$$

Mitigation possible by  
squeezing

# Passive optical clocks: limitations



Accuracy  $< 10^{-18}$

Stability  $\sim 10^{-16}$  @ 1s

Bloom et al, Nature 506, 71 (2014)

Quantum Projection  
Noise (QPN)

$$\sigma_{\text{QPN}}(\tau) = \frac{0.264}{\nu_c T_p} \sqrt{\frac{T_c}{N\tau}}$$

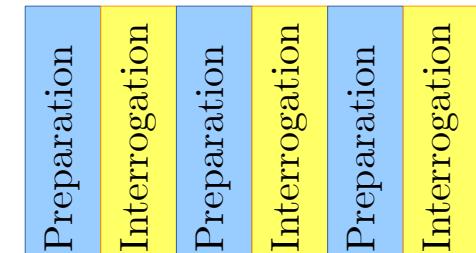
(Rabi interrogation)

$$\sigma_{\text{QPN}} \sim 10^{-17} \tau^{-1/2} \quad (N \sim 5000)$$

Mitigation possible by  
squeezing/QND

Dick effect

Santarelli et al, IEEE Tr 45, 887 (1998)



$$\sigma_{\text{Dick}}^2(\tau) = \frac{1}{\tau} \sum_{n=1}^{\infty} \left| \frac{G(n/T_c)}{G(0)} \right|^2 S_y(n/T_c)$$

$$\sigma_{\text{Dick}} \sim 10^{-16} \tau^{-1/2} \text{ (typ.)}$$

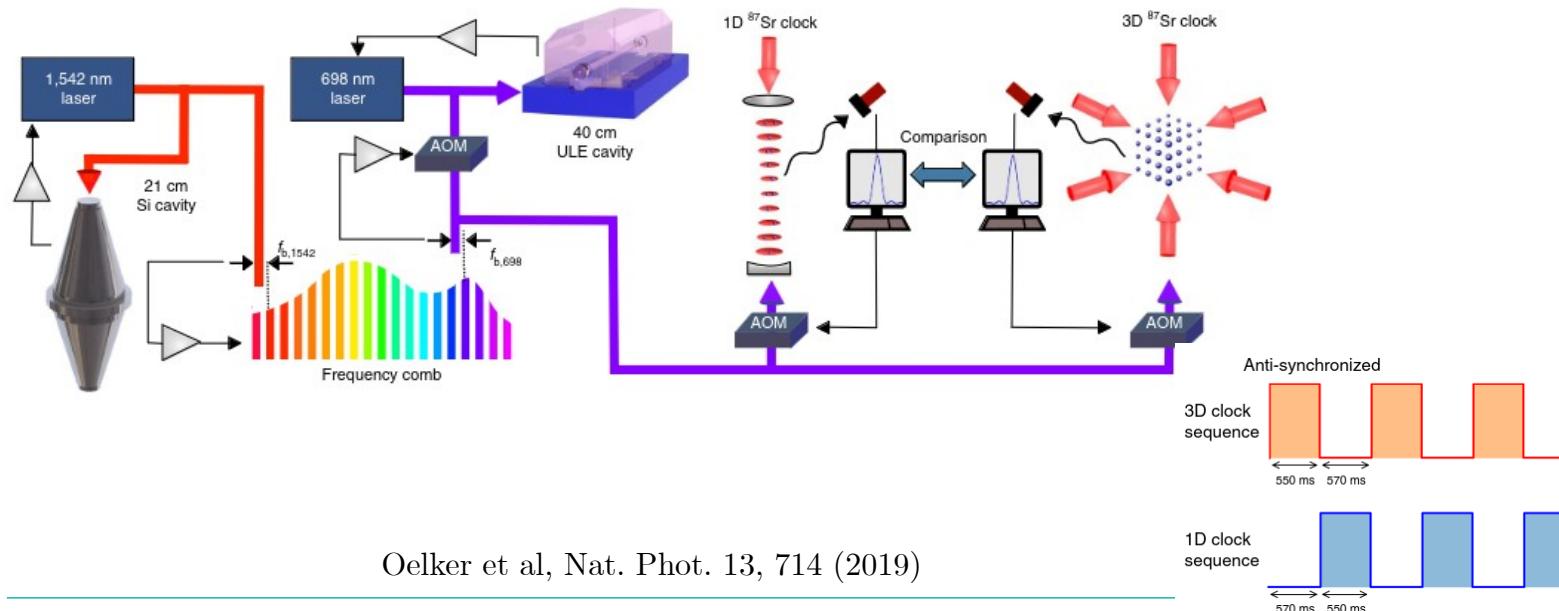
# Zero dead-time measurements



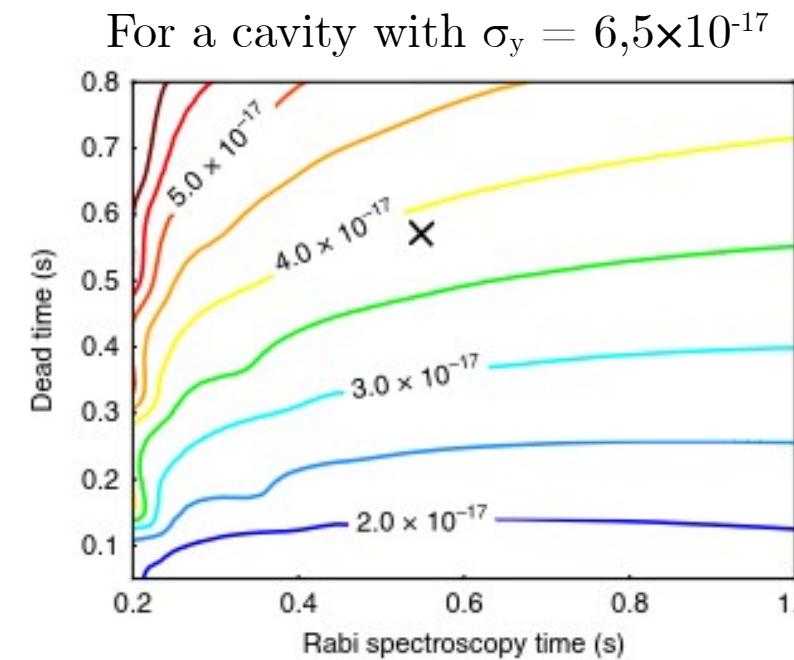
NIST (2019):



## Demonstration of $4.8 \times 10^{-17}$ stability at 1 s for two independent optical clocks



Oelker et al, Nat. Phot. 13, 714 (2019)



# How to build better local oscillators ?



NIST

Now limited by thermal Brownian noise:

→ increase length →  $L = 48$  cm

$$\sigma_y \geq 8 \times 10^{-17} \text{ (Häfner et al, Optica 40, 2112 (2015))}$$

$$S_x(f) = S_x^{spacer}(f) + 2 S_x^{substrate}(f) + 2 S_x^{coat}(f) \propto T,$$

dominated by  $S_x^{coat}(f)$  for silicon spacer

→ use crystalline coatings (Cole et al, Nat. Phot. 7, 644 (2013))

$$S_x^{coat}(f) \rightarrow S_x^{coat}(f) / 10$$

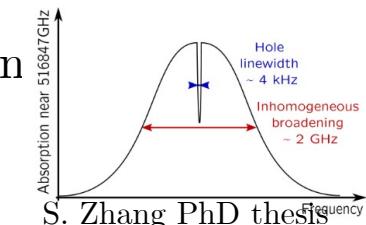
→ reduce T → put cavities in cryostats

(4 K, 16-18 K, 124 K reported, prospects for 100 mK)

$$\sigma_y \geq 3,5 \times 10^{-17} \text{ (21 cm, 124 K, crystalline coatings)}$$

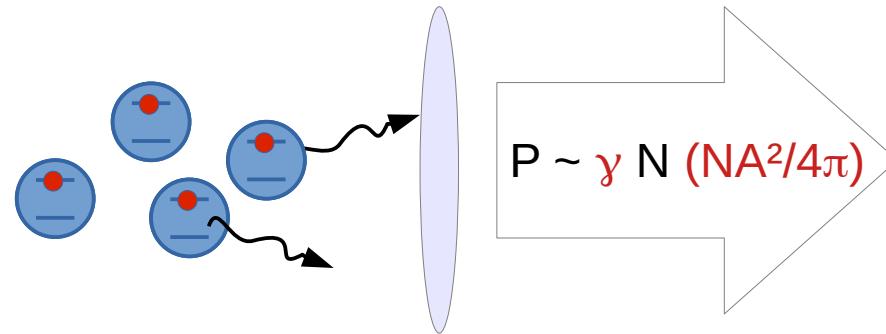
But: - no 50 cm long cavity in a cryostat...  
- cryostat = hardly compatible with (future) tran

Spectral Hole Burning = promising, but still a cryostat



# Replacing the clock laser by a fluorescing source

Collect fluorescence of “static” atoms on a narrow line

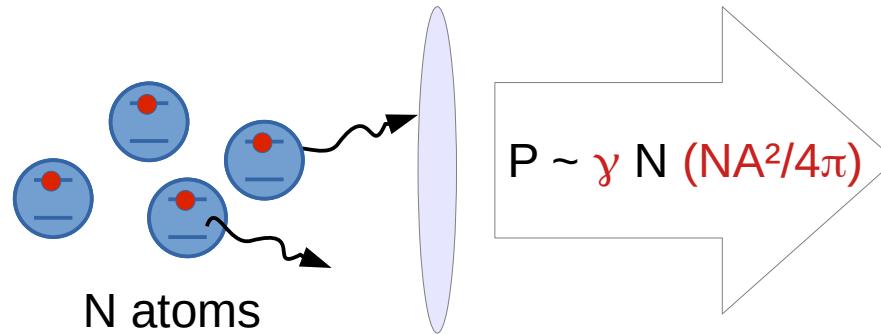


light **continuously** self-referenced  
to a narrow atomic transition

*Collecting a useful amount of light ?*

# Replacing the clock laser by a fluorescing source

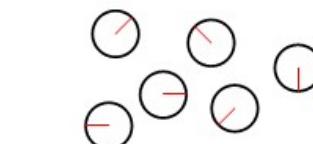
Collect fluorescence of “static” atoms on a narrow line



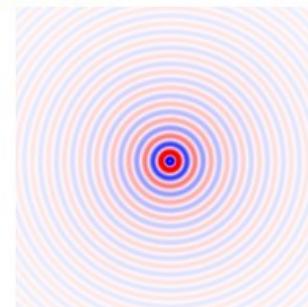
light **continuously** self-referenced  
to a narrow atomic transition

*Collecting a useful amount of light ?*

Emitting dipoles with  
random phases

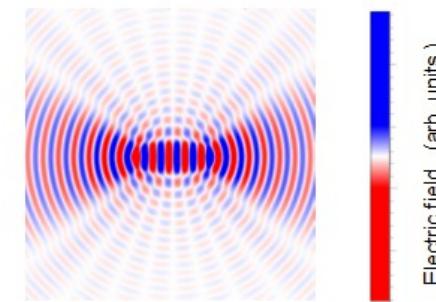
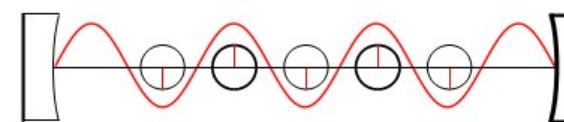


Electric field



Random interference  
Power  $\sim N$

**Synchronized phases**  
along a well defined optical mode



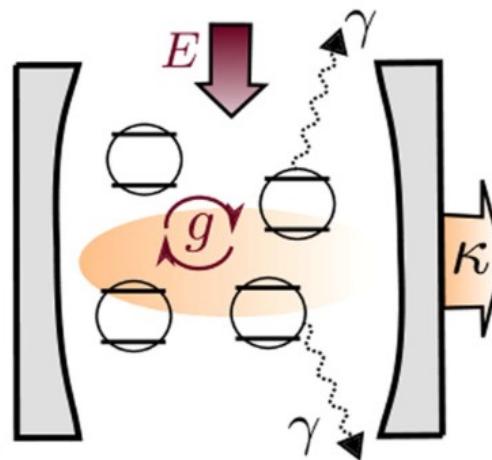
**Constructive interference ( $P \sim N^2$ )**  
into a **directional mode**

**Superradiance : a regime where  
the dipoles self-synchronize**

The cavity plays a lesser role than  
in standard lasers  
(mostly: defines the mode)

# The superradiant laser

Mode defined by an optical cavity



$\kappa$ : optical mode lifetime

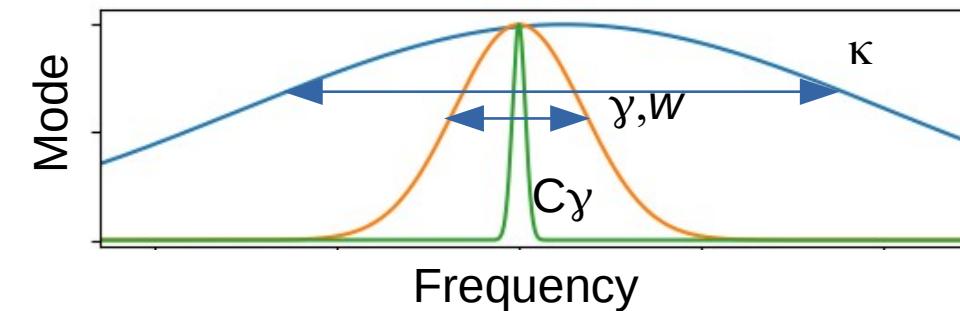
$\gamma$  : atom linewidth

$g$  : atom-photon coupling

## Bad cavity limit:

$\kappa \gg \gamma$  and the various broadening mechanisms  $w$ :

→ radiation frequency robust to cavity length fluctuations



Cavity pulling resilience  $\sim w / \kappa$

## Weak coupling regime : $g^2/\kappa < \gamma$

(Cooperativity  $C = g^2/\kappa\gamma < 1$ )

→  $g^2/\kappa = C\gamma$  = spontaneous emission rate *to the mode*  
*(Purcell rate)*  
~ linewidth of the superradiant laser

# Reaching the superradiant regime

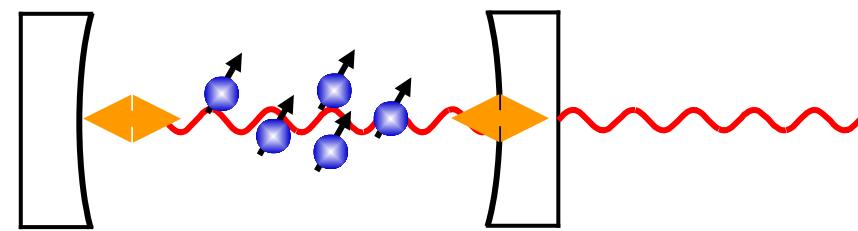
Quick photon leakage ( $\kappa > \sqrt{N} g$ )

the electric field  $E$  follows the total atomic dipole  $D$

$$E(t) \sim D(t)$$

- One excited atom (dipole  $\langle d \rangle = 0$ ) is driven by  $H = -d \cdot E$  towards the collective dipole  $D$  of the other atoms
  - will then emit in phase with the others

For  $N g^2/\kappa \gg \gamma, \omega$ ,  
 $(NC > 1)$ ,  
a macroscopic atomic dipole  
becomes stationary



Much less photons than atoms in the cavity

Coherence in the collective atomic dipole

Linewidth can be down to the Purcell rate  $\frac{4g^2}{\kappa} \ll \kappa/N_{photons}$

Foundational proposals:

Meiser, Holland, PRL 102, 163601 (2009): *Prospects for a Millihertz-Linewidth Laser*

JB Chen 2009, Chinese Science Bulletin 54, 348 (2009) : *Active optical clock*

# A steady-state superradiant laser with less than one intracavity photon

Justin G. Bohnet<sup>1</sup>, Zilong Chen<sup>1</sup>, Joshua M. Weiner<sup>1</sup>, Dominic Meiser<sup>1†</sup>, Murray J. Holland<sup>1</sup> & James K. Thompson<sup>1</sup>

## PHYSICAL REVIEW LETTERS

Highlights    Recent    Accepted    Collections    Authors    Referees    Search    Press    About    Editorial Team

### Subnatural Linewidth Superradiant Lasing with Cold $^{88}\text{Sr}$ Atoms

Sofus Laguna Kristensen, Eliot Bohr, Julian Robinson-Tait, Tanya Zelevinsky, Jan W. Thomsen, and Jörg Helge Müller

Phys. Rev. Lett. **130**, 223402 – Published 31 May 2023

## PHYSICAL REVIEW X

Highlights    Recent    Subjects    Accepted    Collections    Authors    Referees    Search    Press

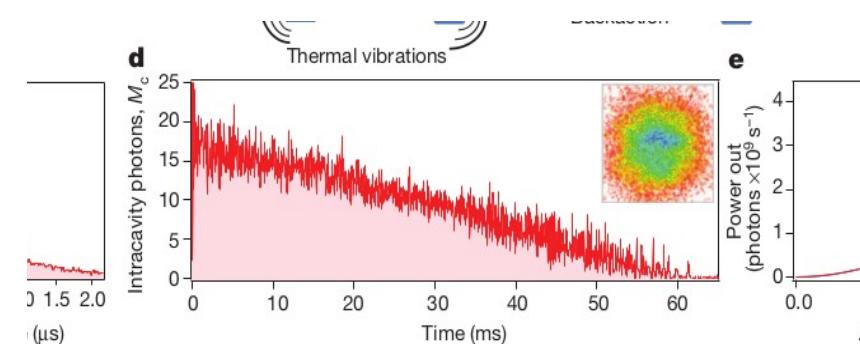
Open Access

### Frequency Measurements of Superradiance from the Strontium Clock Transition

Matthew A. Norcia, Julia R. K. Cline, Juan A. Muniz, John M. Robinson, Ross B. Hutson, Akihisa Goban, G. Edward Marti, Jun Ye, and James K. Thompson

Phys. Rev. X **8**, 021036 – Published 9 May 2018

But : always a finite-duration pulse



Other achievements e.g. in the Tomsen, Hemmerich, and Schreck groups

# Current challenge: true continuous operation

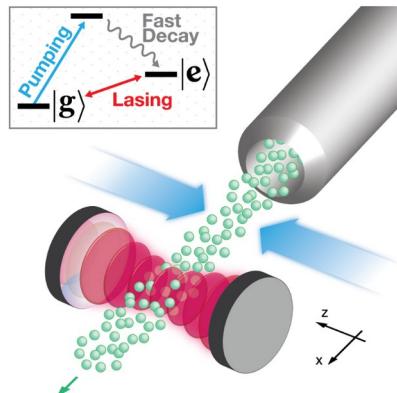


Quasi-steady state: can be achieved by repumping (JILA) or by seeding atoms from a metastable state (Copenhagen)

BUT: finite lifetime of the atoms

→ how to bring in new atoms?

(Thermal) Beam operation



Chen, Chin. Sci. Bull. 54, 348 (2009)  
Liu et al., PRL 125, 253602 (2020)

$$N_{\text{th}} = 2/(C \gamma T_2)$$

- Reloading of (cold) atoms
- ring cavity system (JILA)
  - cold atomic beam (Amsterdam?)
  - refill from MOT (Hamburg)
  - sequential reloading (FEMTO-ST)



# Current challenge: true continuous operation

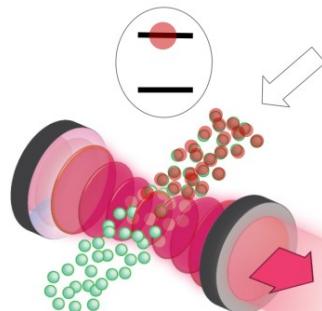


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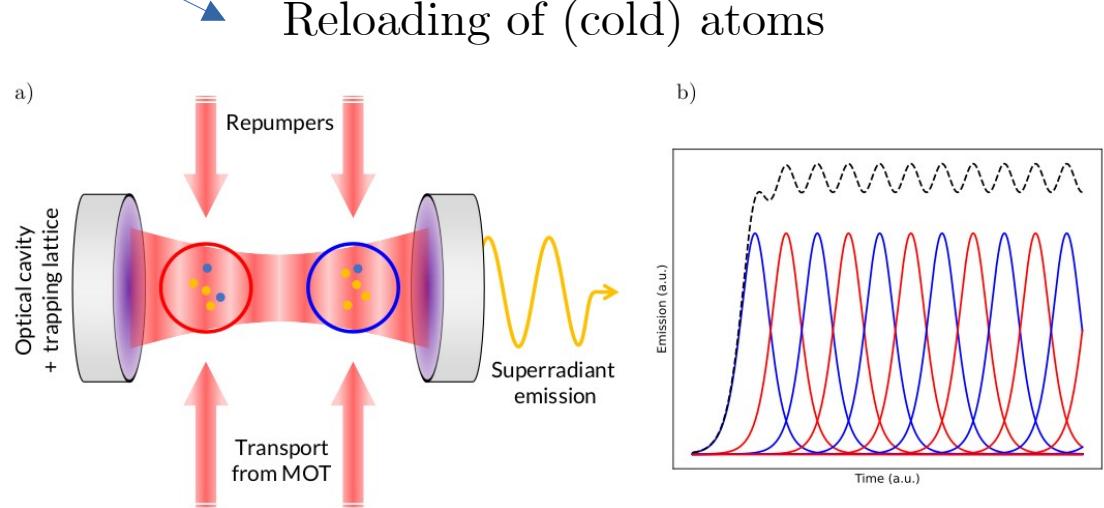
(Thermal) Beam operation



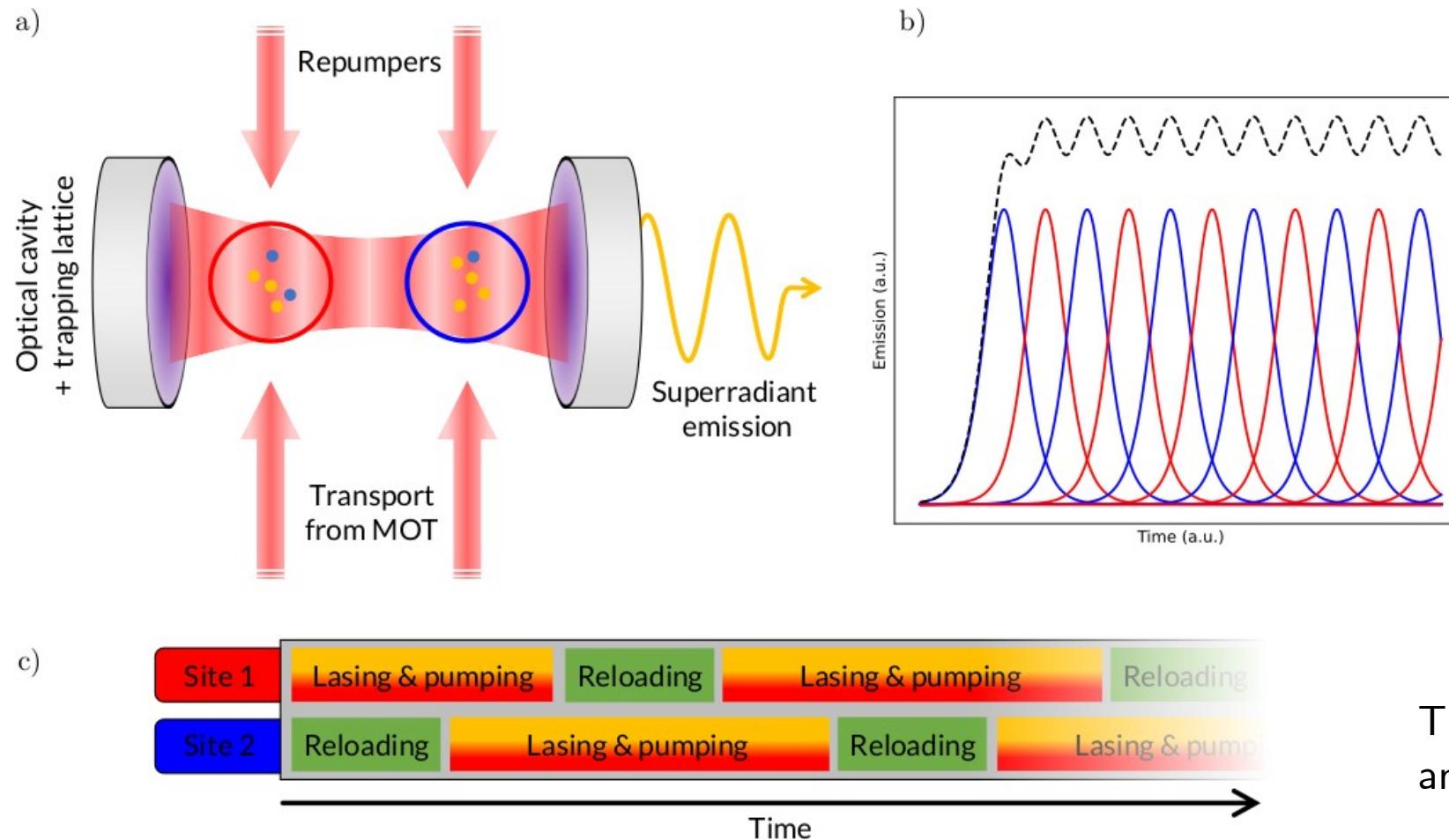
Insertion rate:  $\Gamma$   
Transit rate:  $\Gamma_R = \Gamma/N$

LPL

FEMTO-ST



# FEMTO-ST Apparatus: cold ytterbium atoms with repumping and sequential reloading



Theory by Jana El Badawi  
and Bruno Bellomo

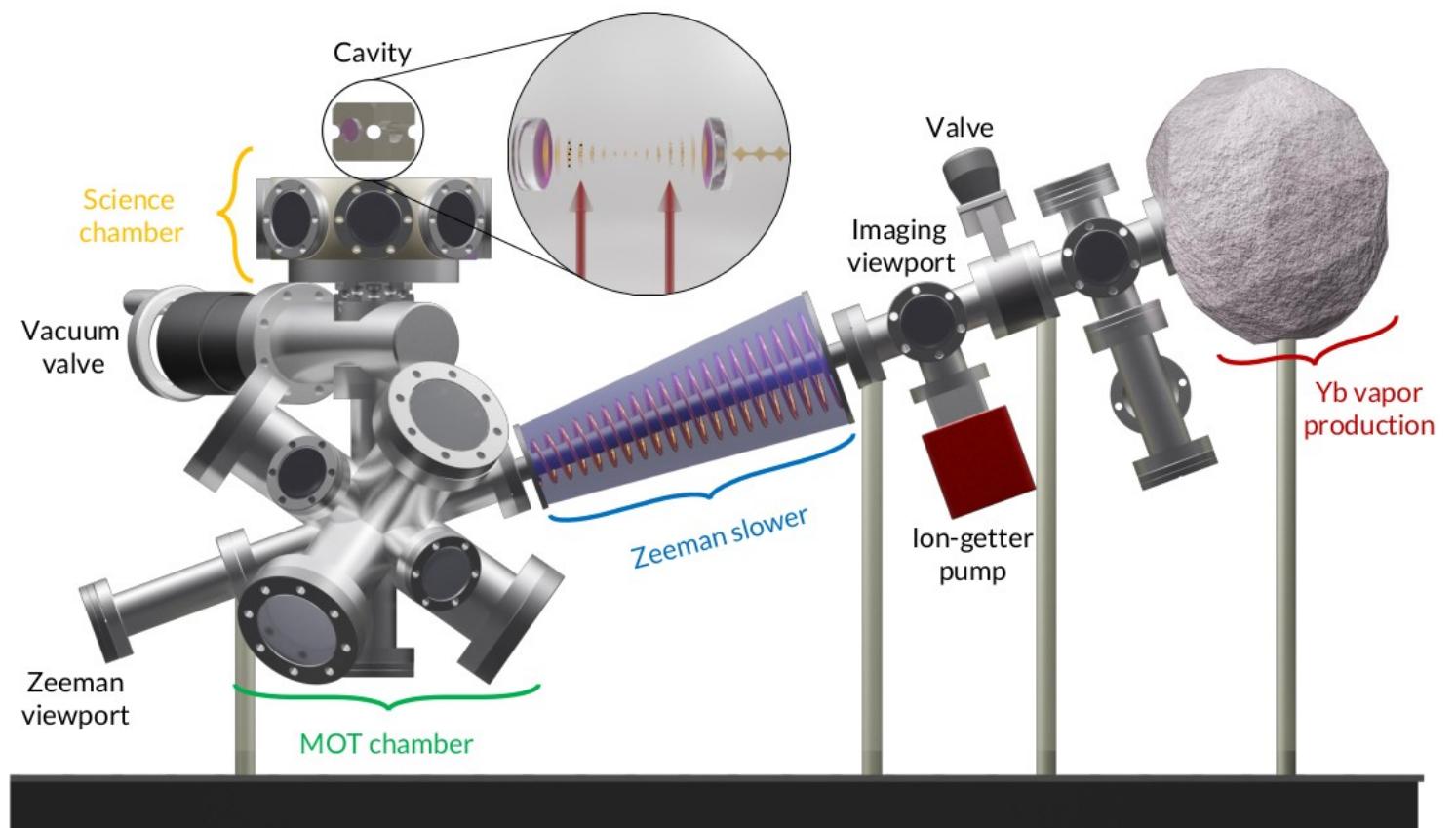
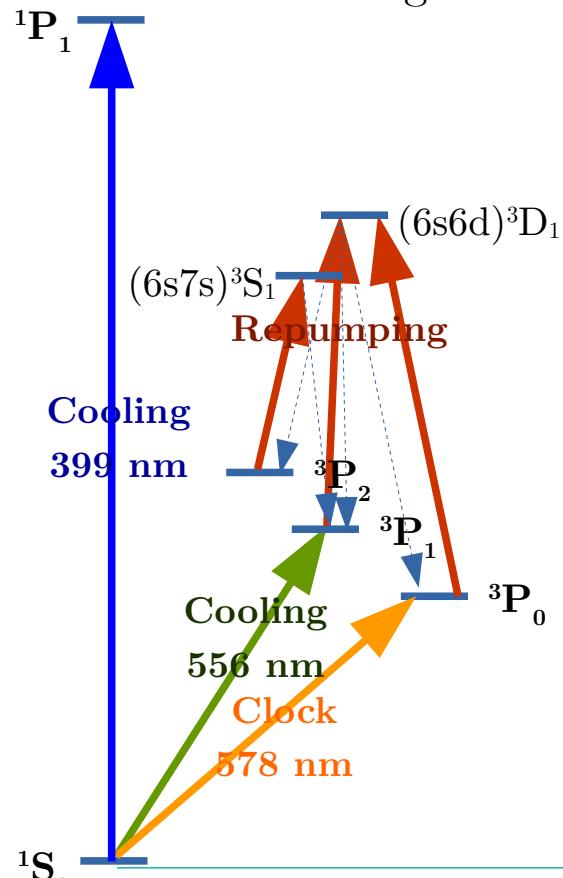
# FEMTO-ST apparatus

$$N_{\text{th}} = 2/(C \gamma T_2)$$

$^{171}\text{Yb}$ : 7 mHz wide  $^1\text{S}_0 \rightarrow ^3\text{P}_0$  transition: threshold atom number  $7\times$  smaller than  $^{87}\text{Sr}$

$^{171}\text{Yb}$  :  $I = 1/2$  : reduced scattering when repumping wrt  $^{87}\text{Sr}$

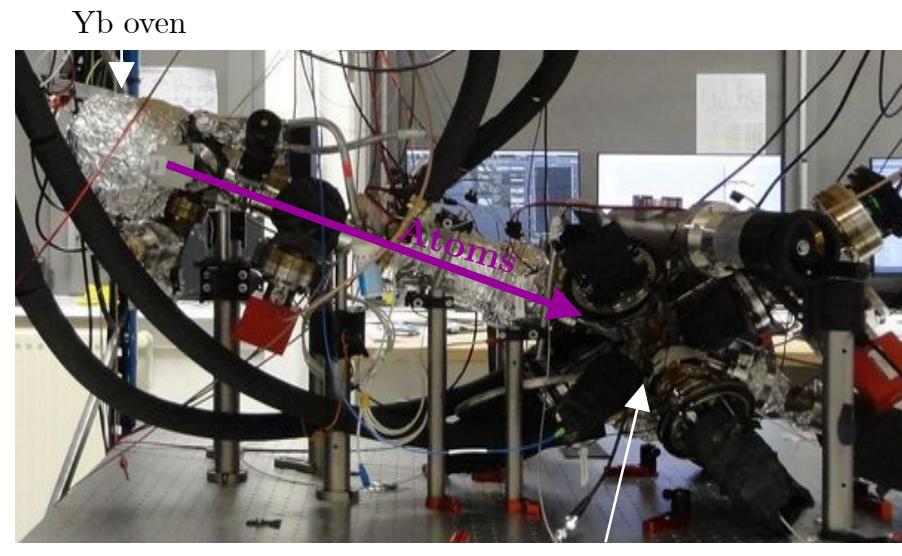
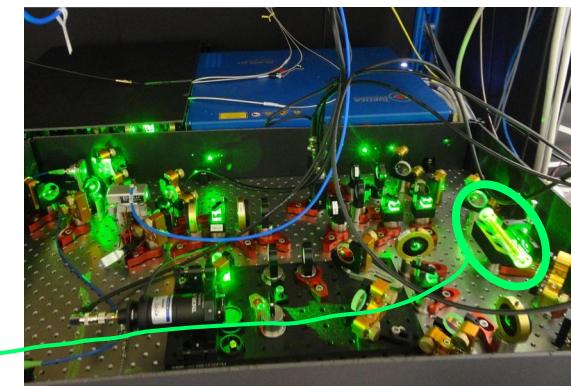
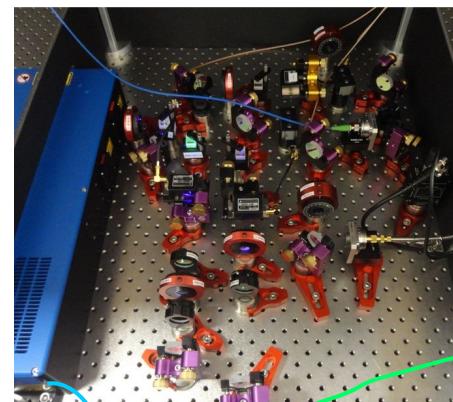
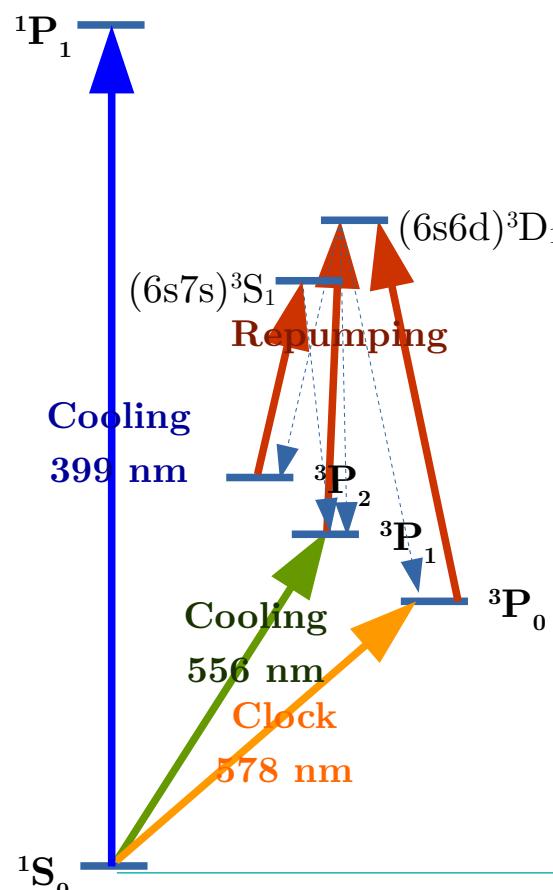
But no straightforward reservoir in  $^3\text{P}_2$  (antitrapped by magic lattice)



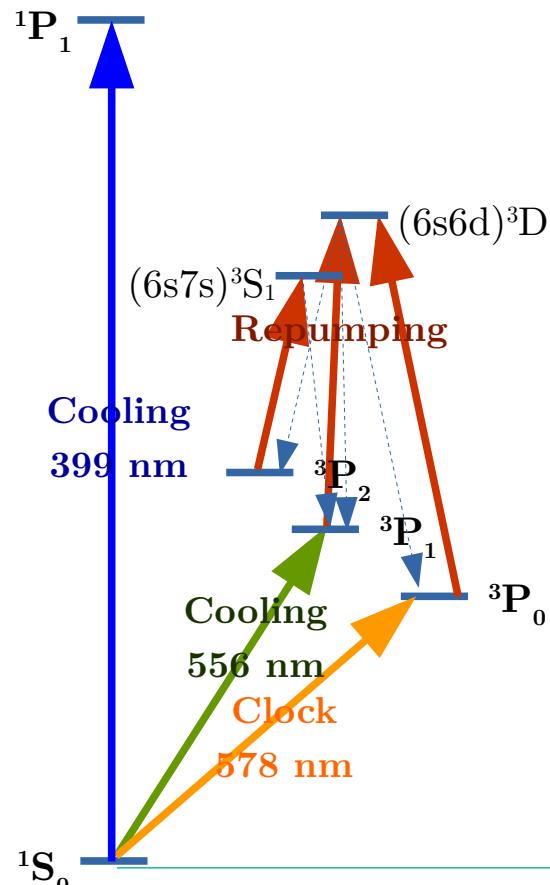
# FEMTO-ST apparatus



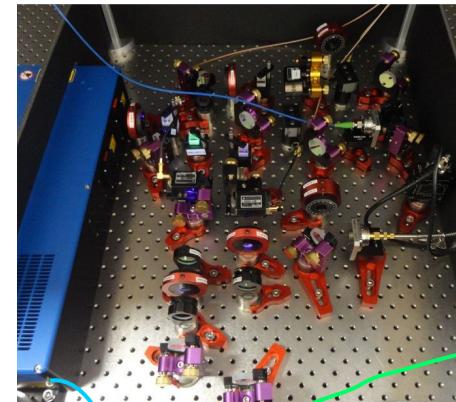
ATOMS



# FEMTO-ST apparatus



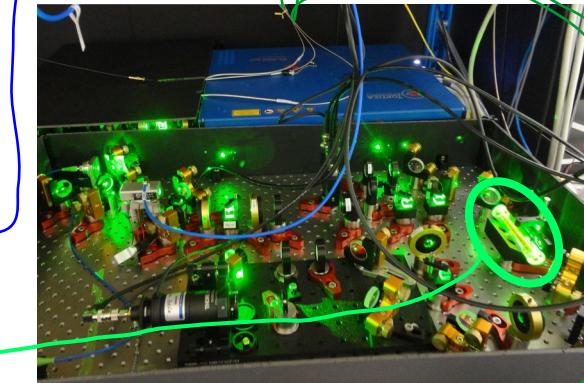
ATOMS



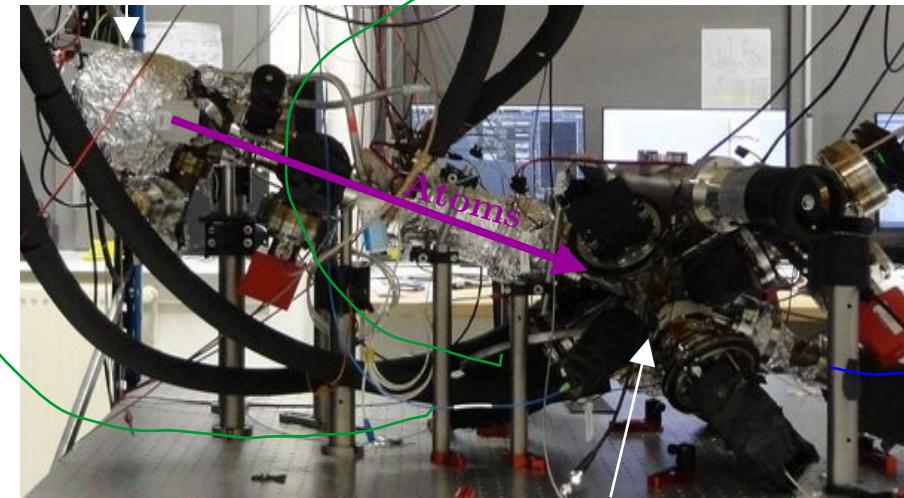
Zeeman

slower

3 retro-reflected MOT beams



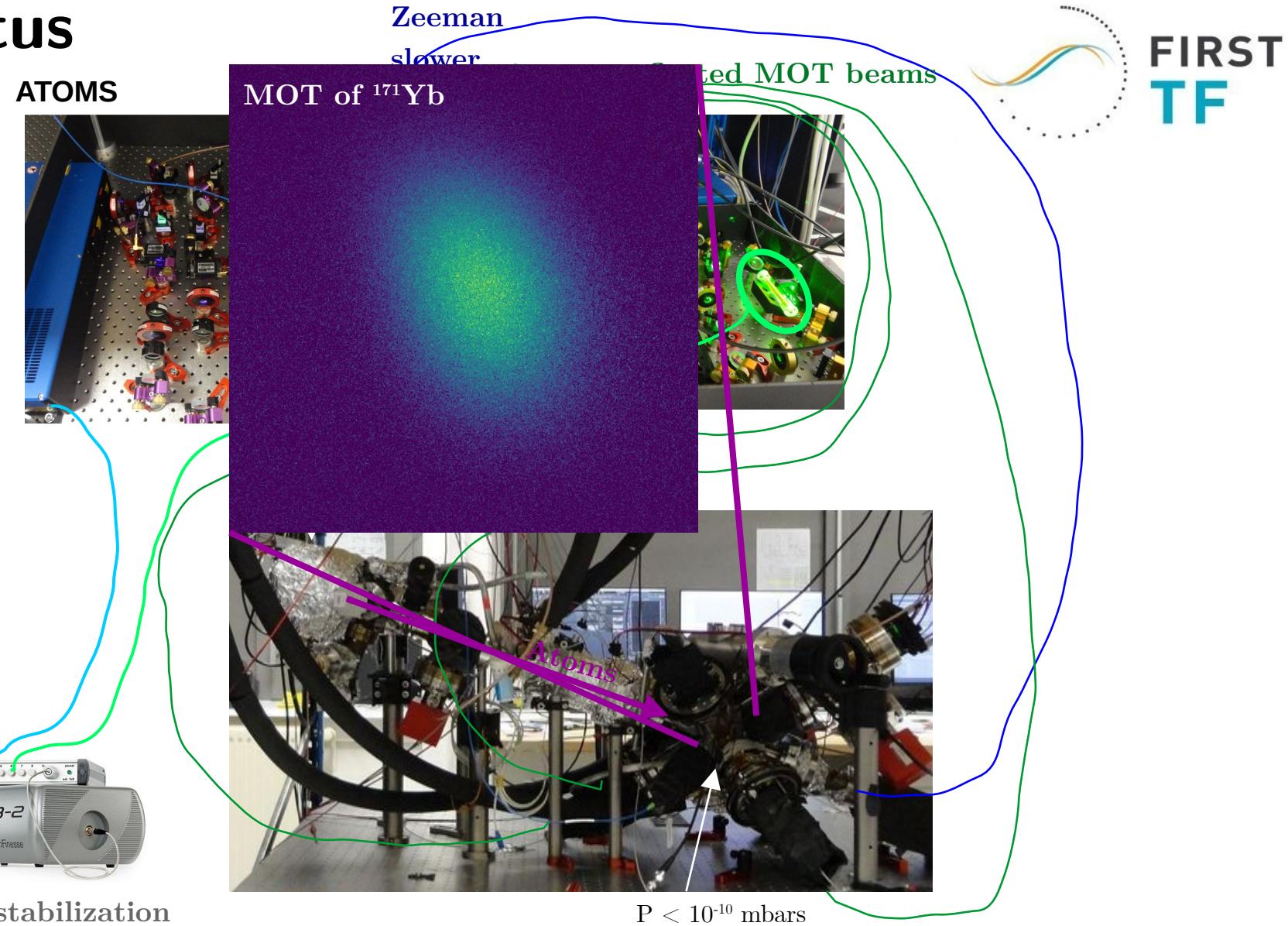
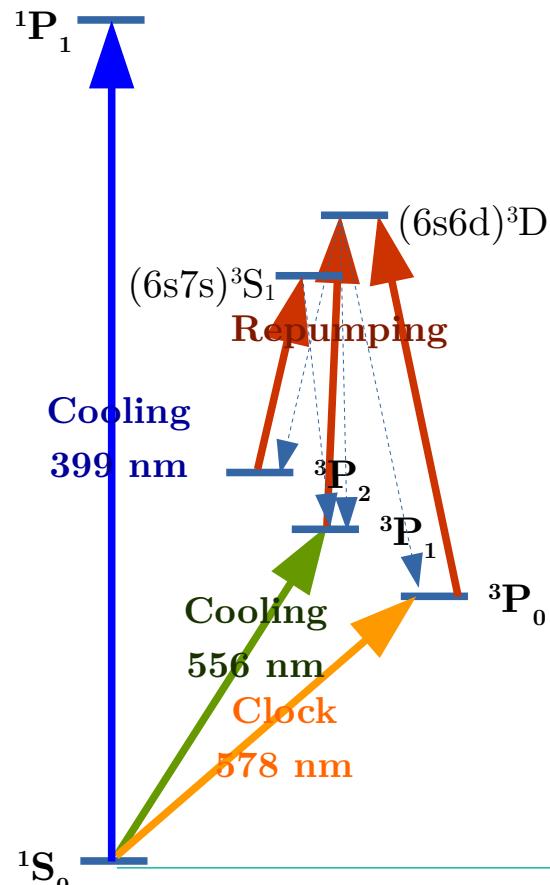
Yb oven



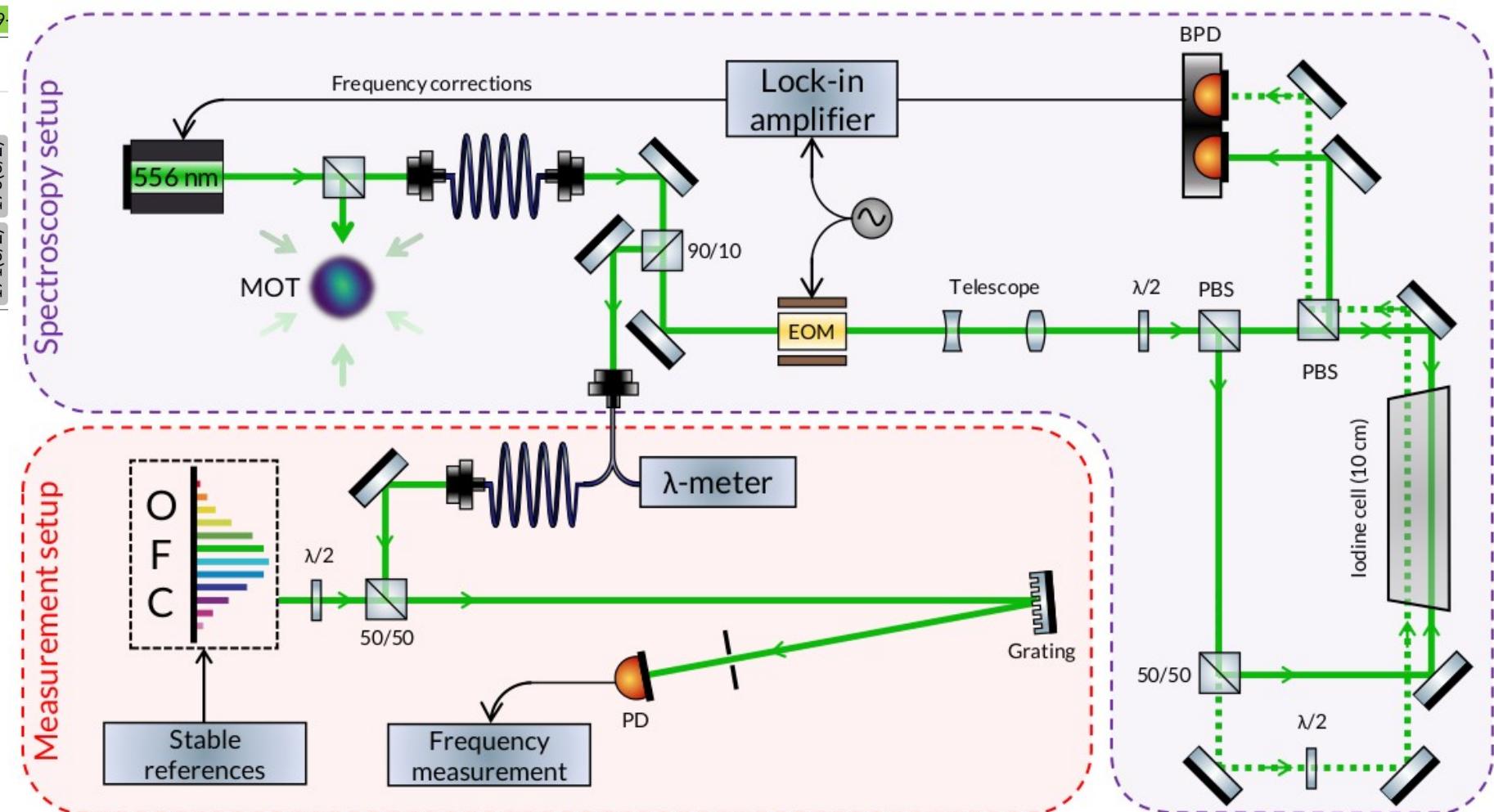
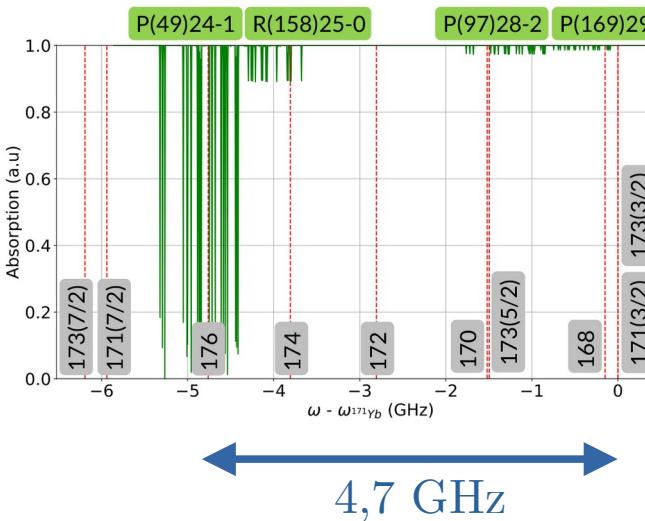
Freq. stabilization

## 3. FEMTO-ST

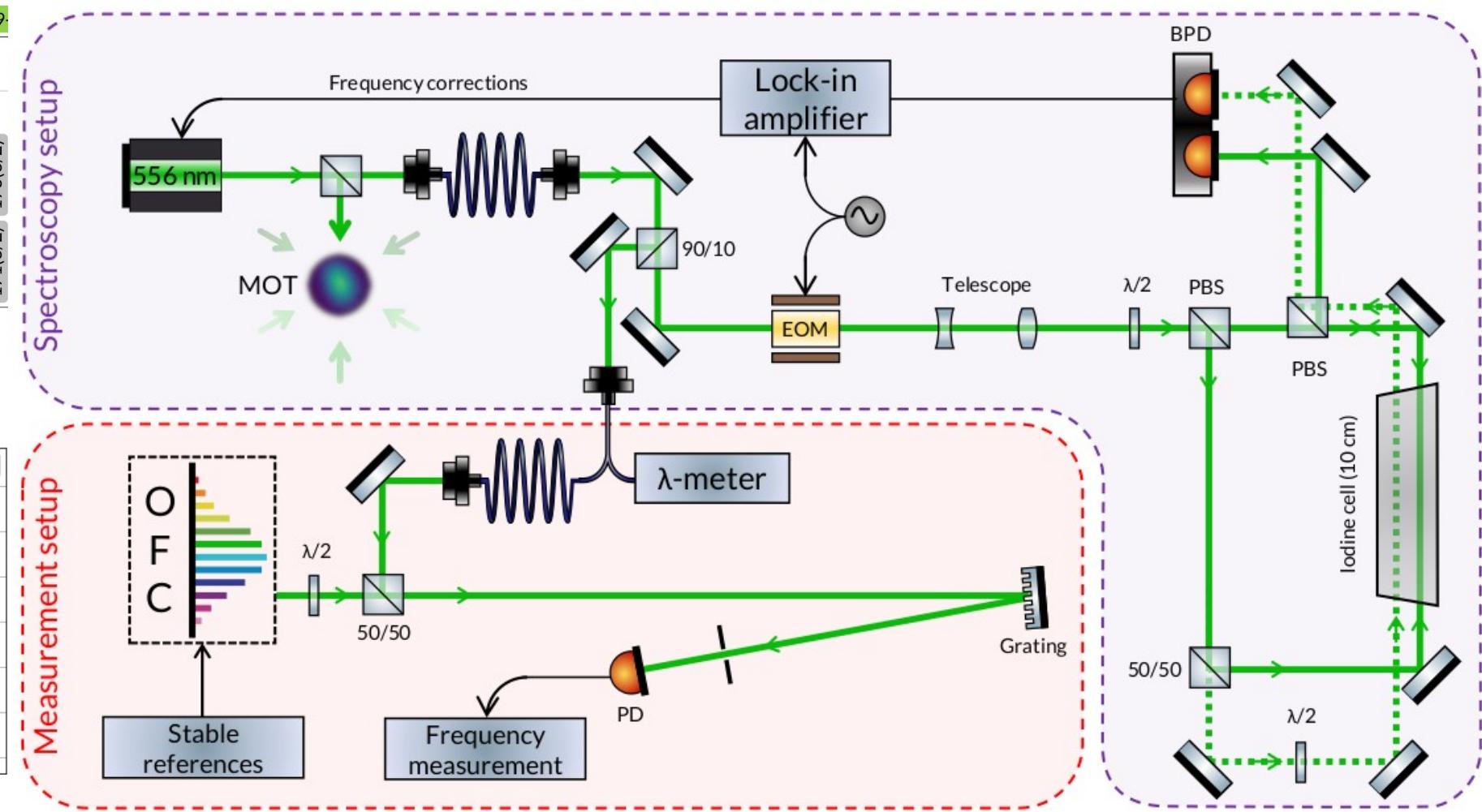
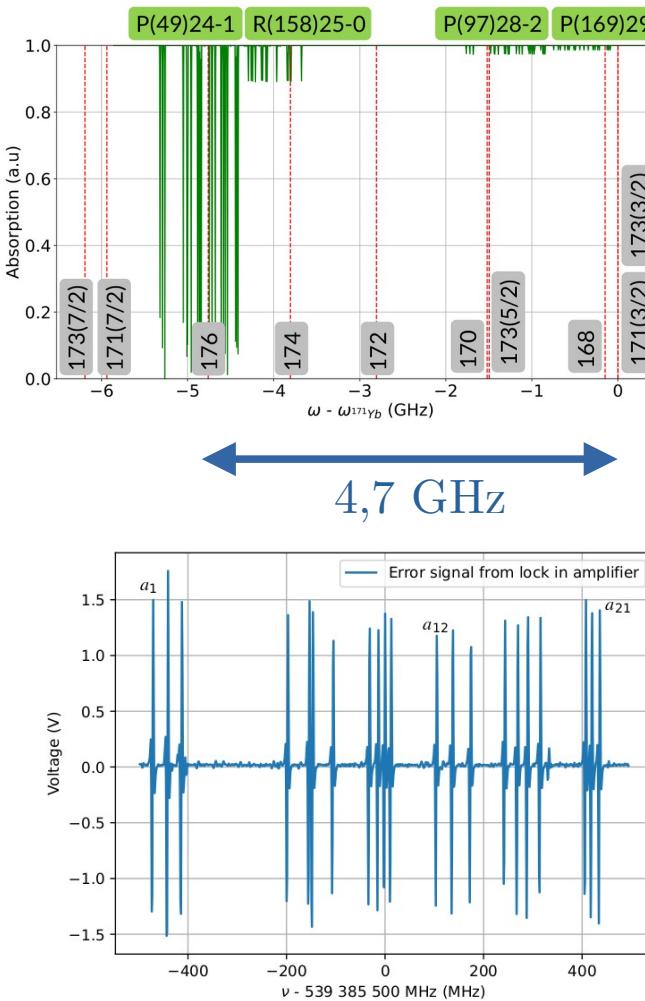
# FEMTO-ST apparatus



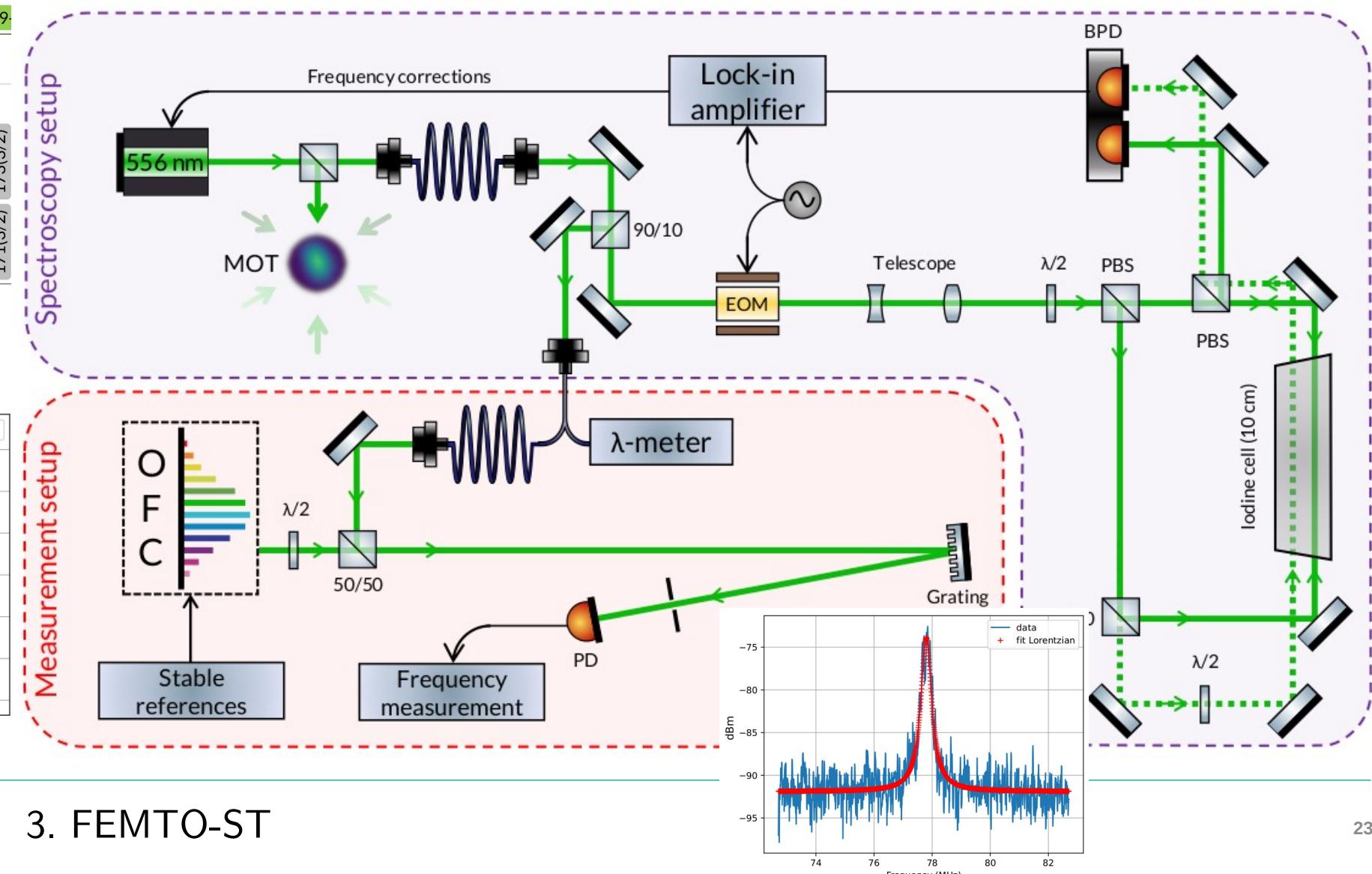
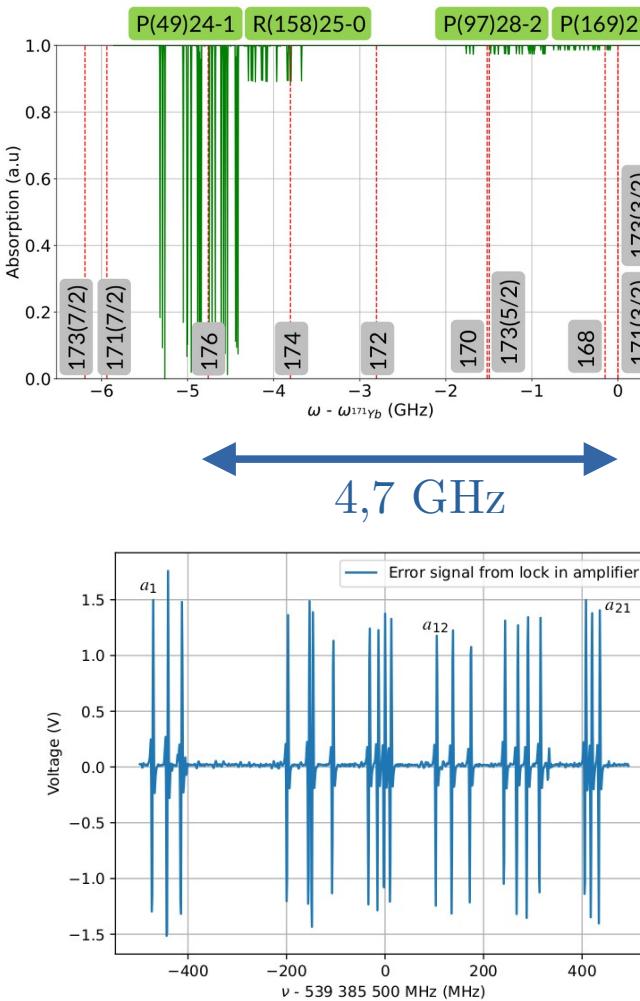
# Iodine measurements



# Iodine measurements

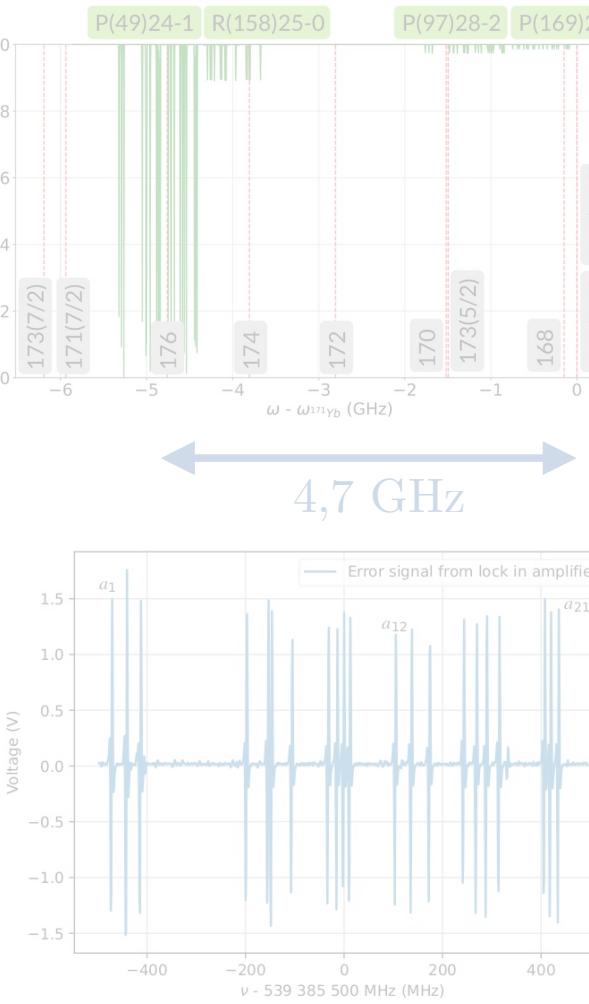
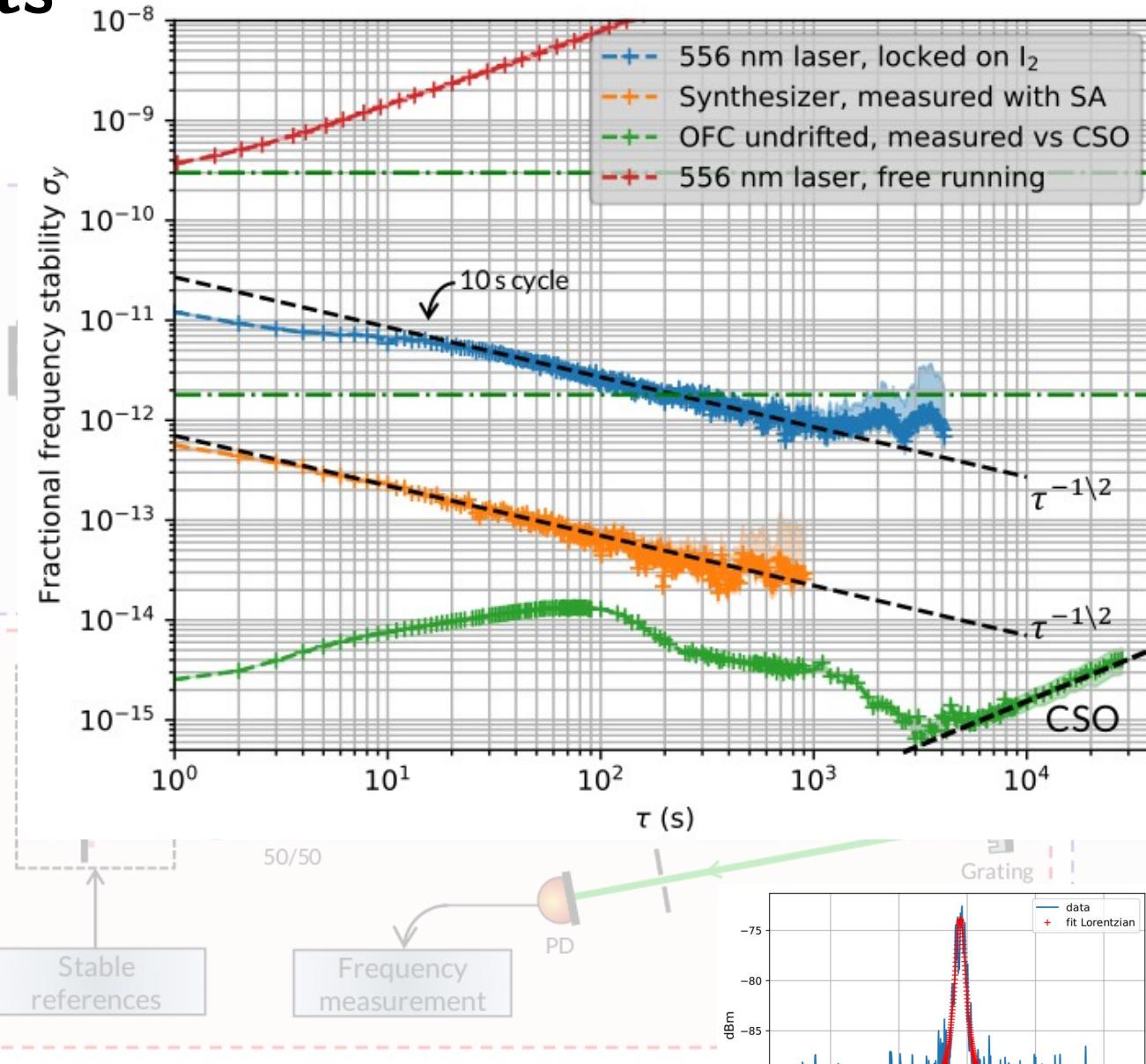
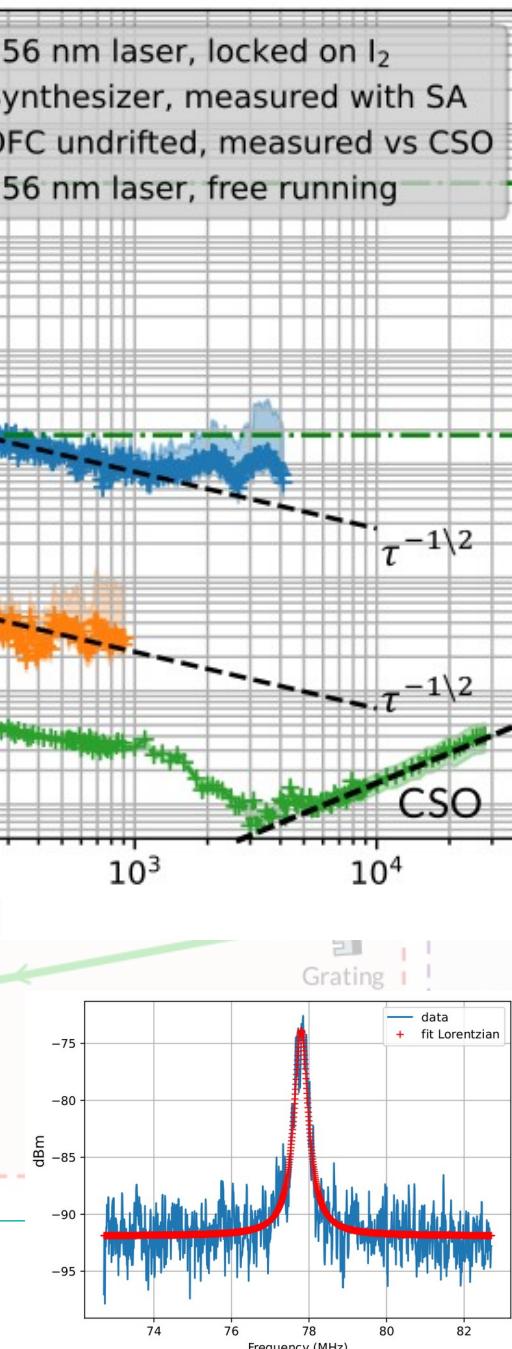
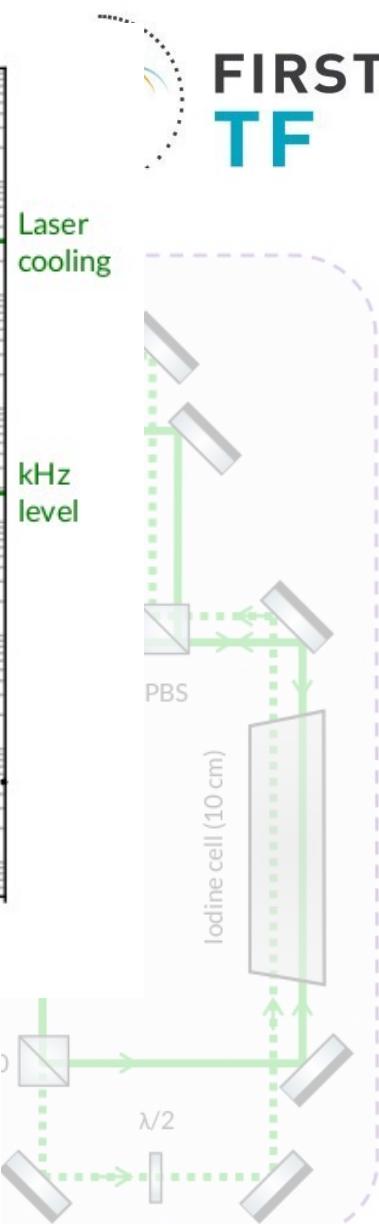


# Iodine measurements



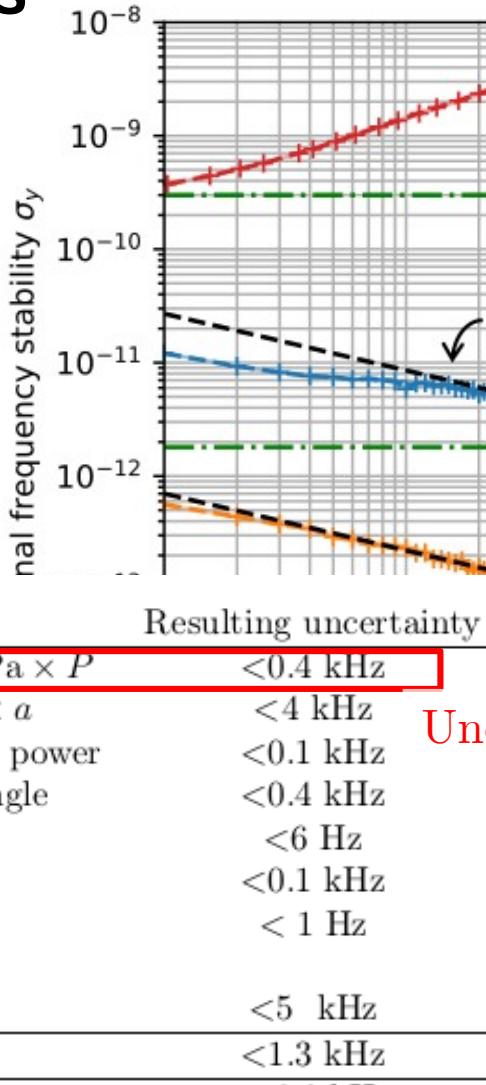
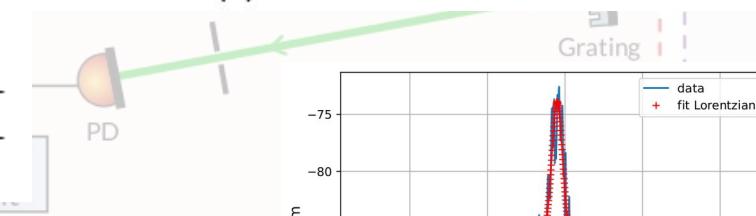
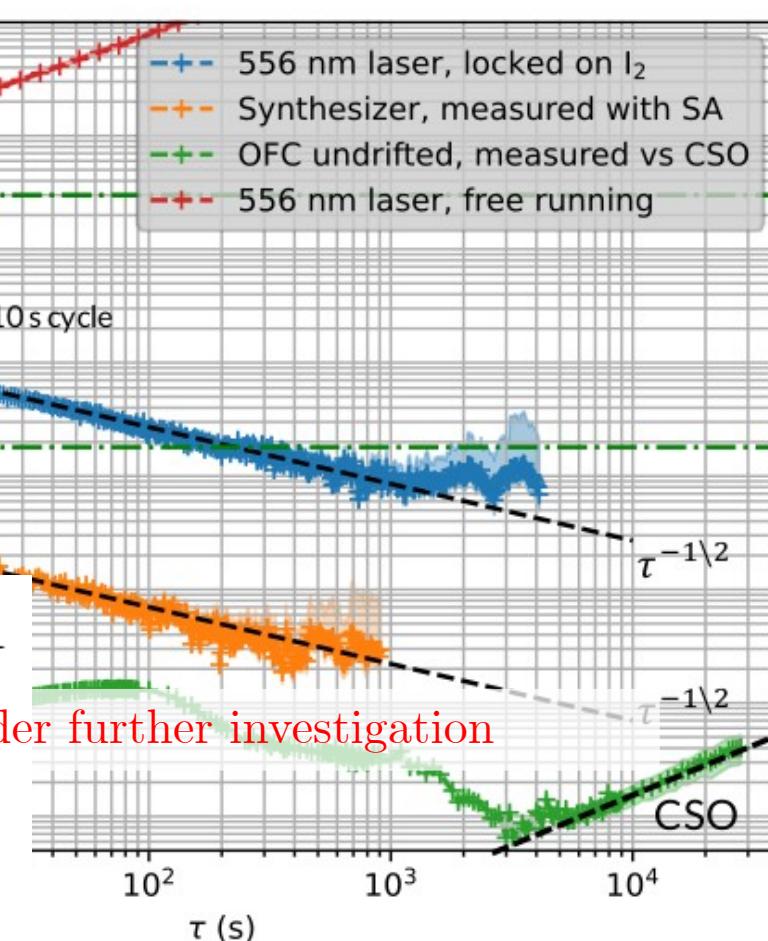
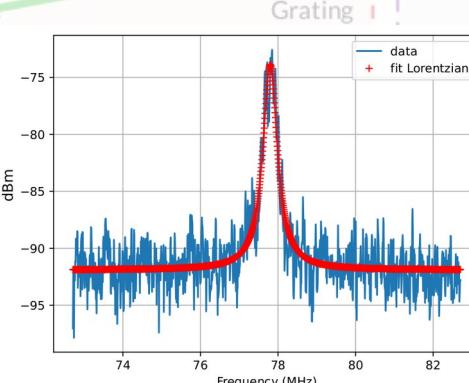
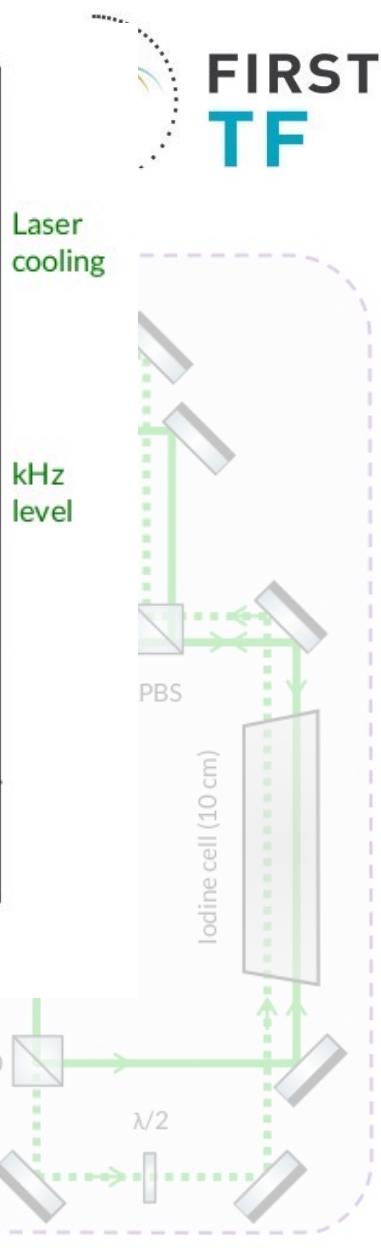
# Iodine measurements

FIRST  
TF

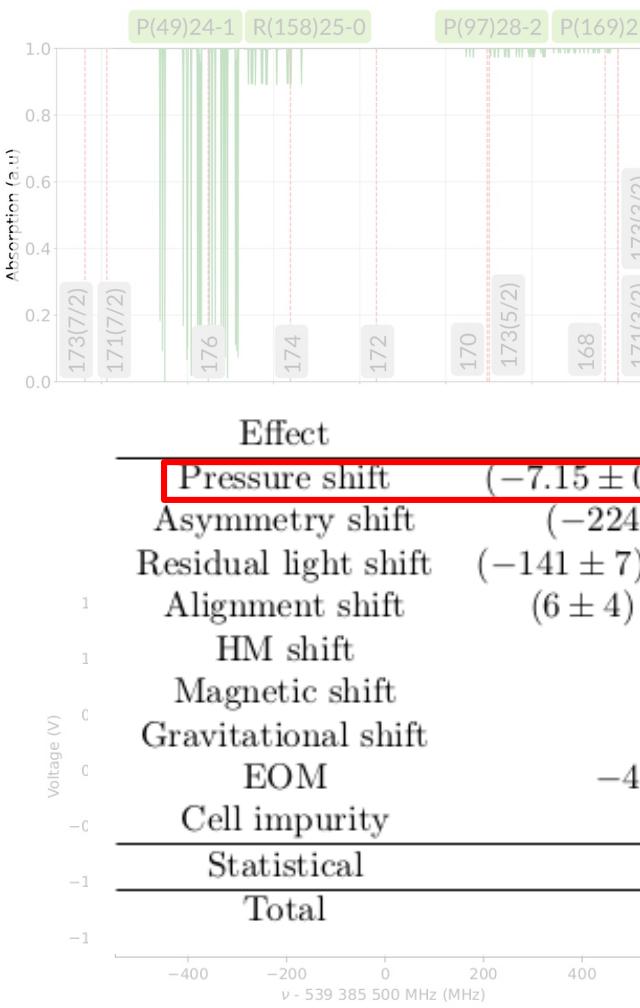


# Iodine measurements

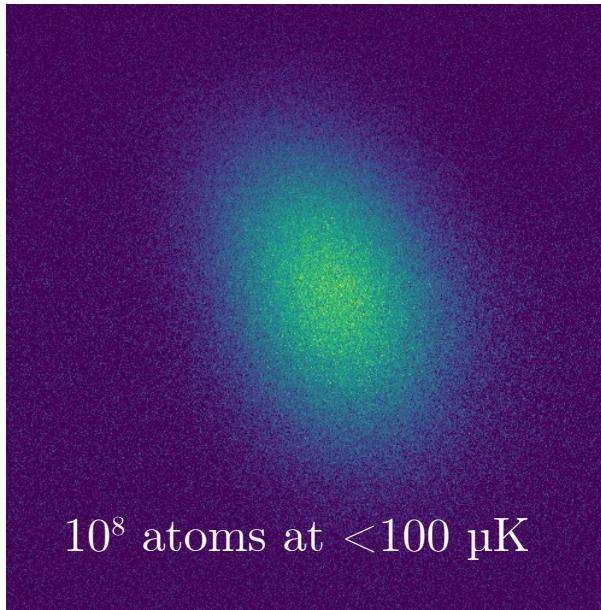
FIRST  
TF



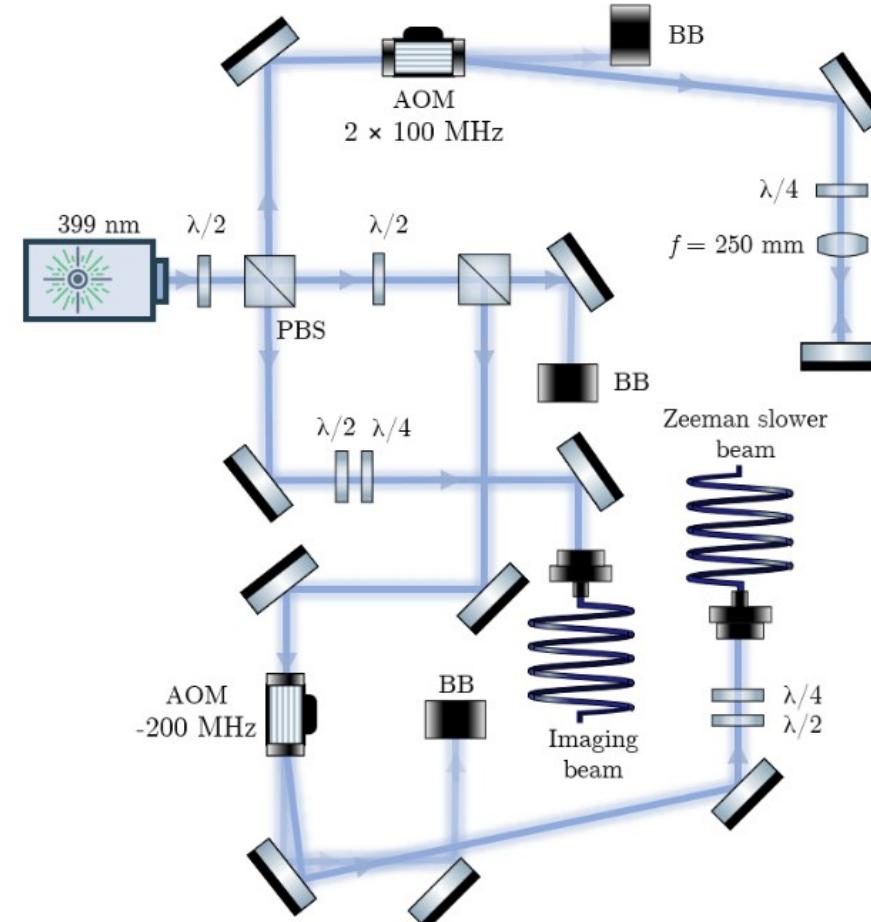
Spectroscopy setup



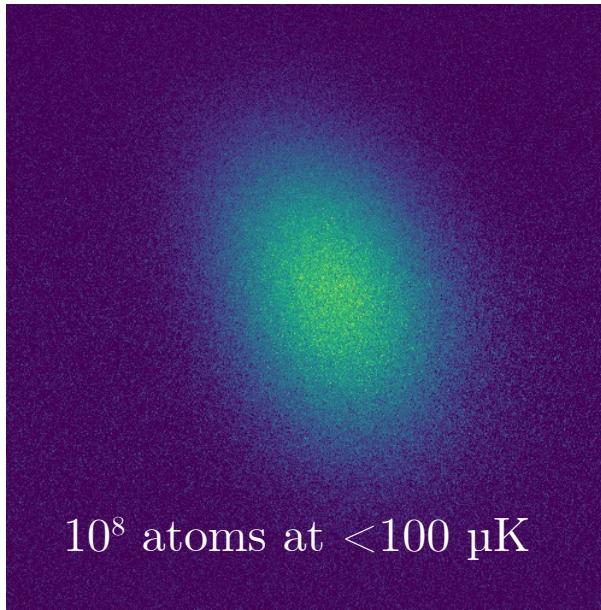
# Ytterbium ensemble



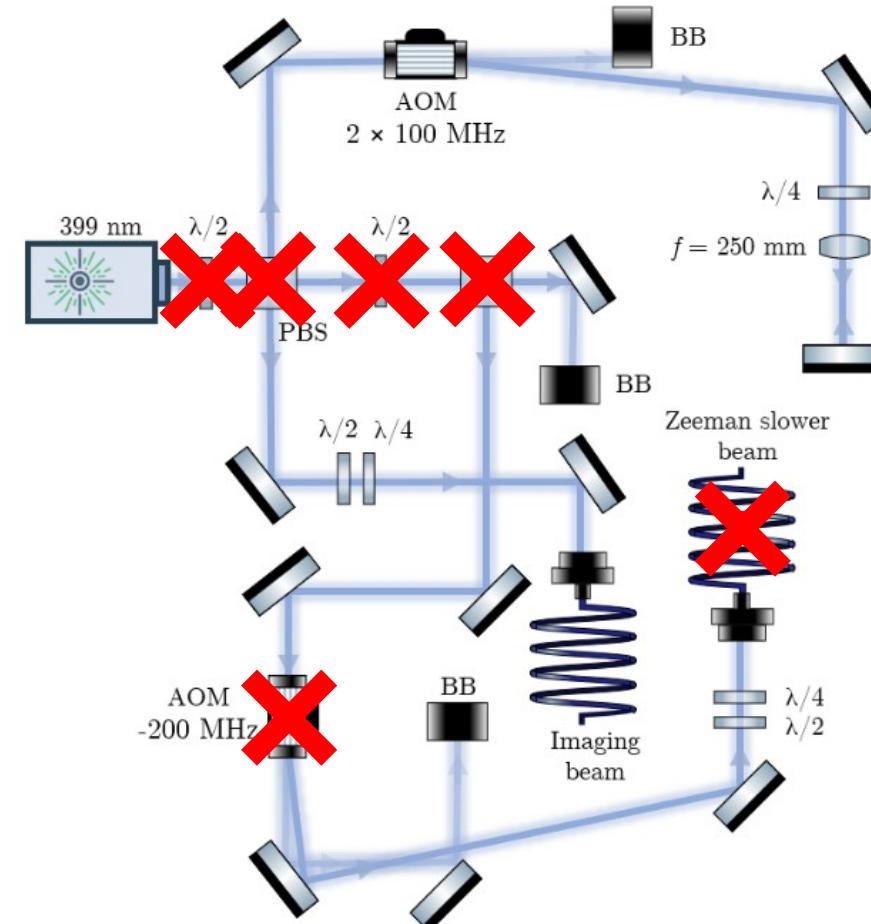
but ...



# Ytterbium ensemble

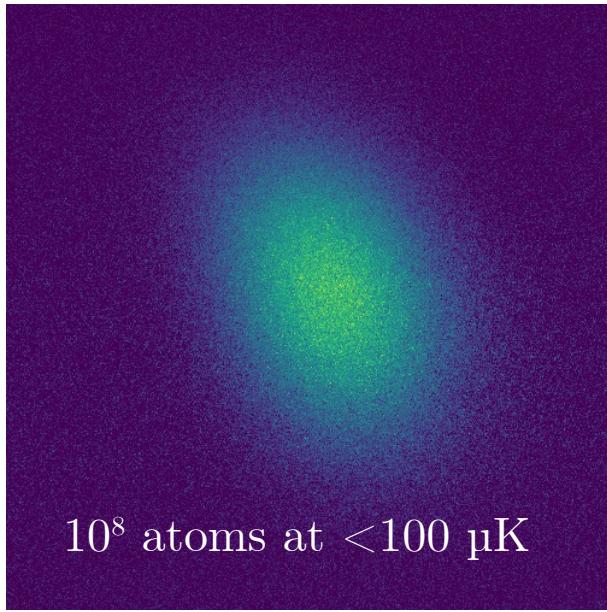


but ...



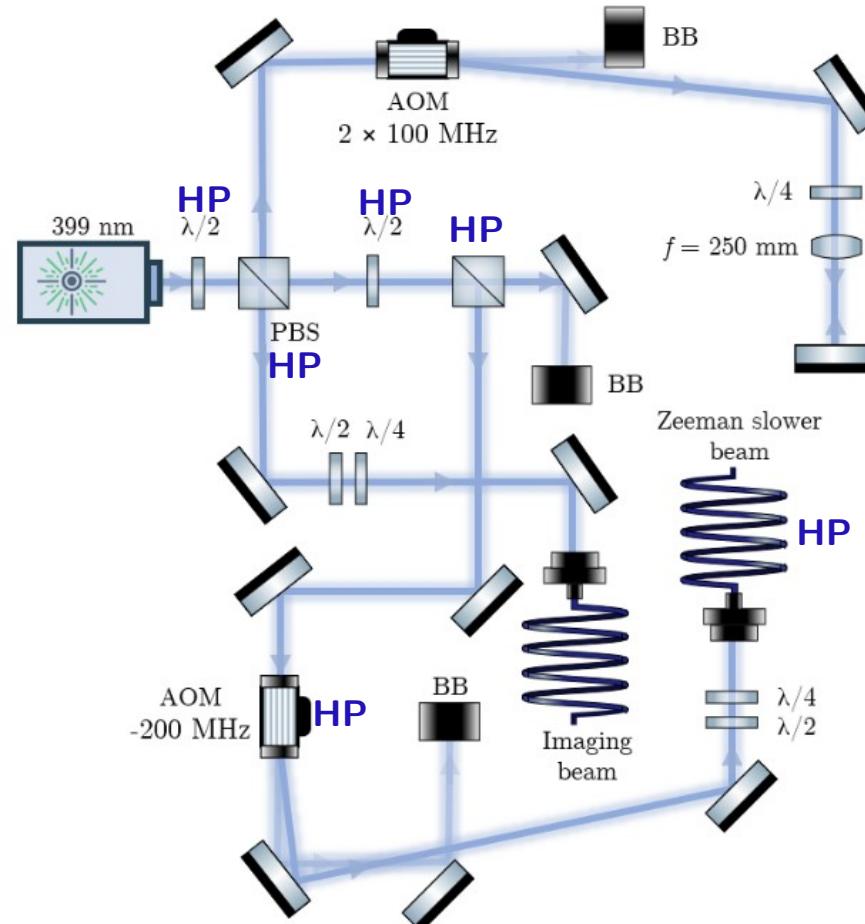
and then (Zeeman viewport) ...

# Ytterbium ensemble

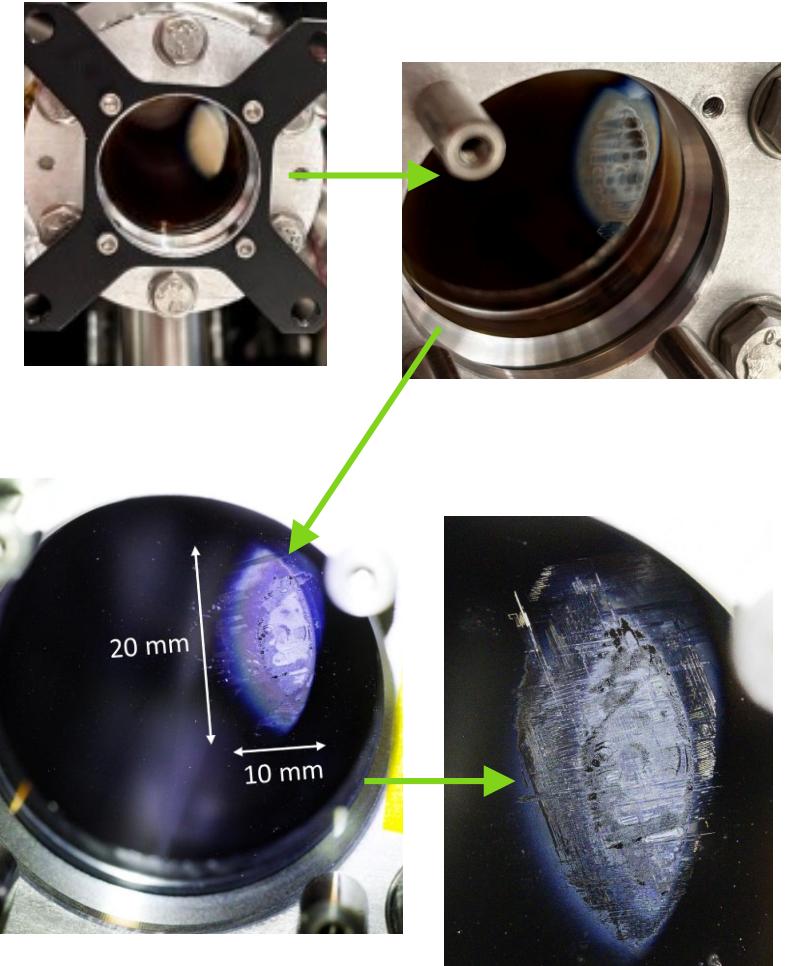


$10^8$  atoms at  $<100 \mu\text{K}$

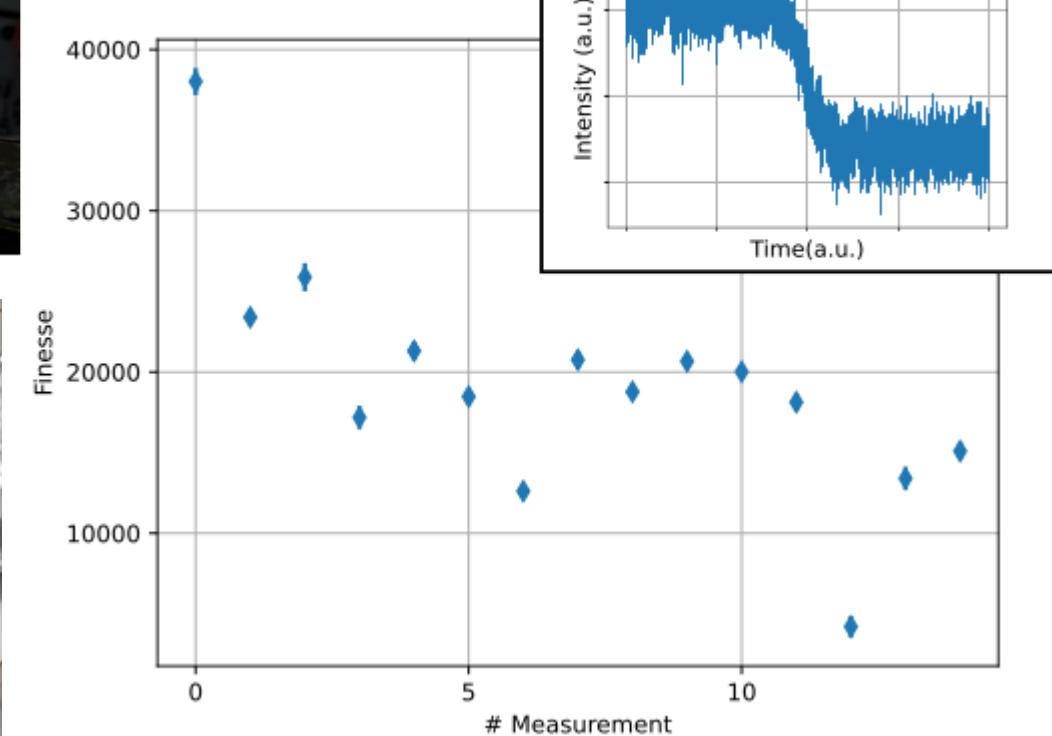
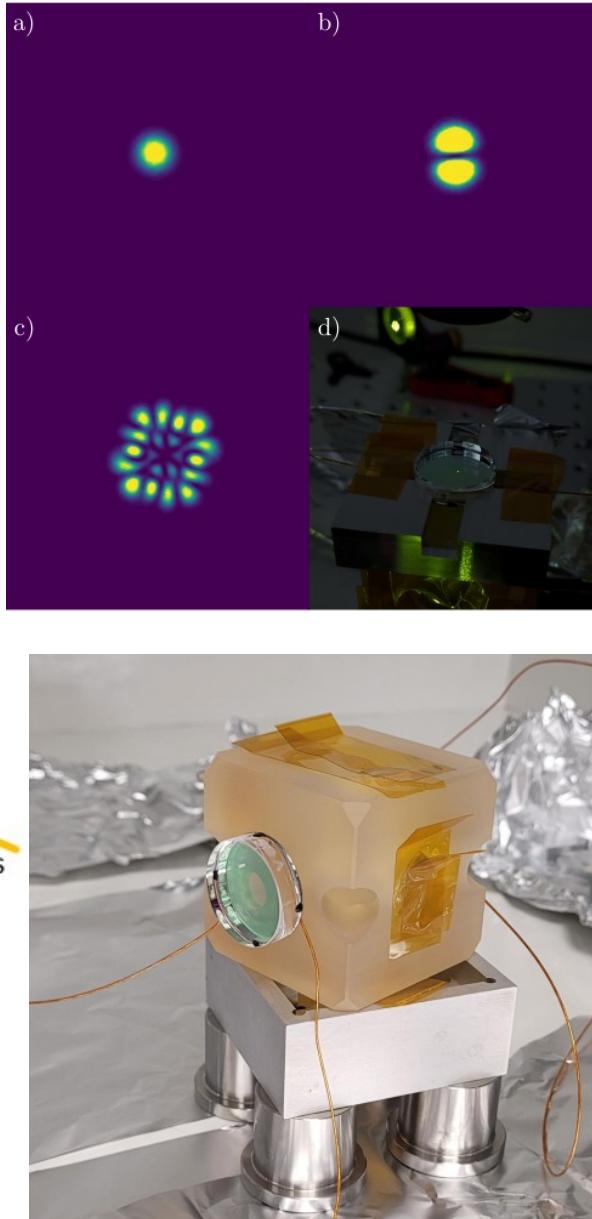
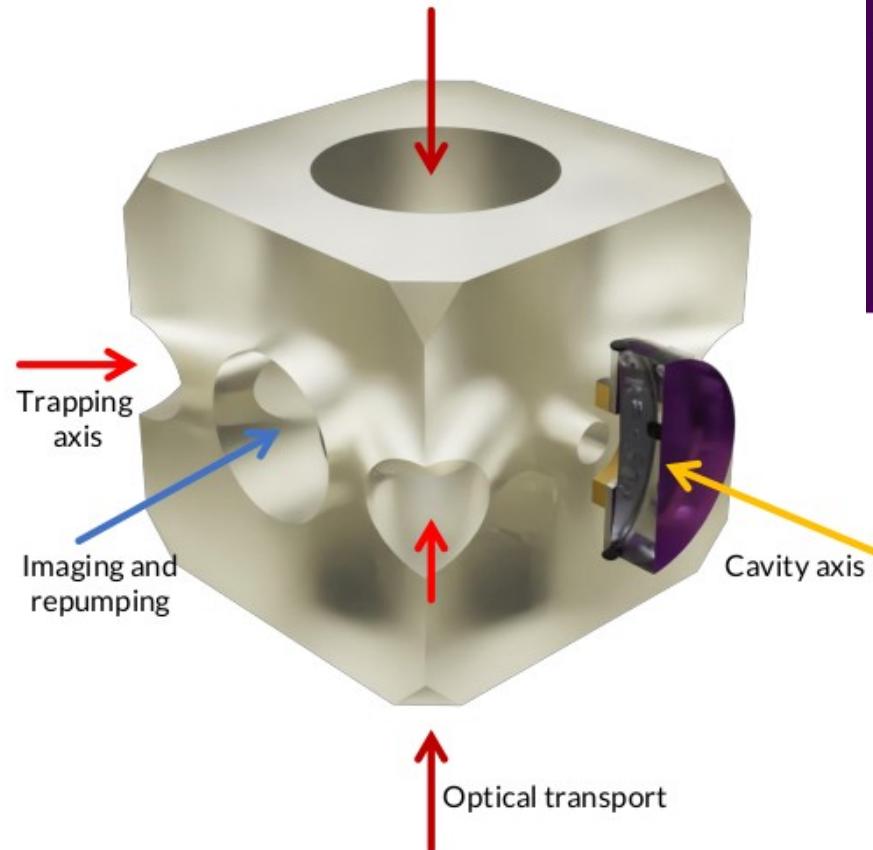
but ...



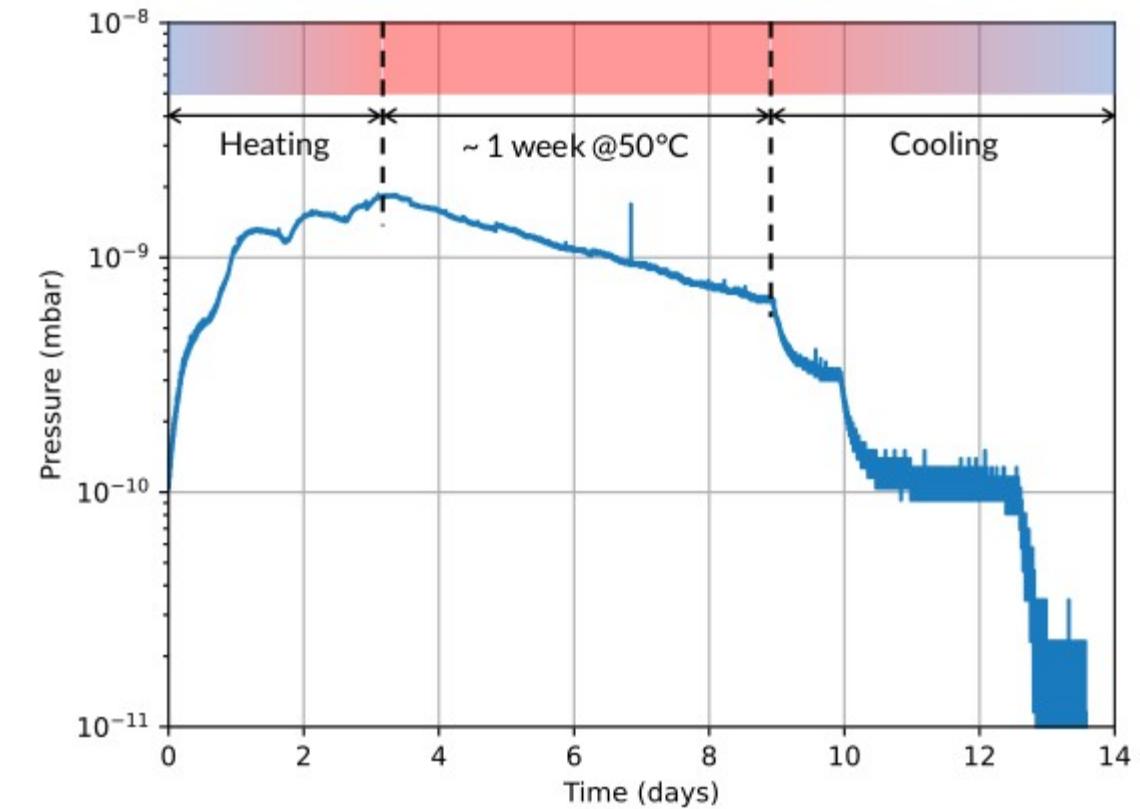
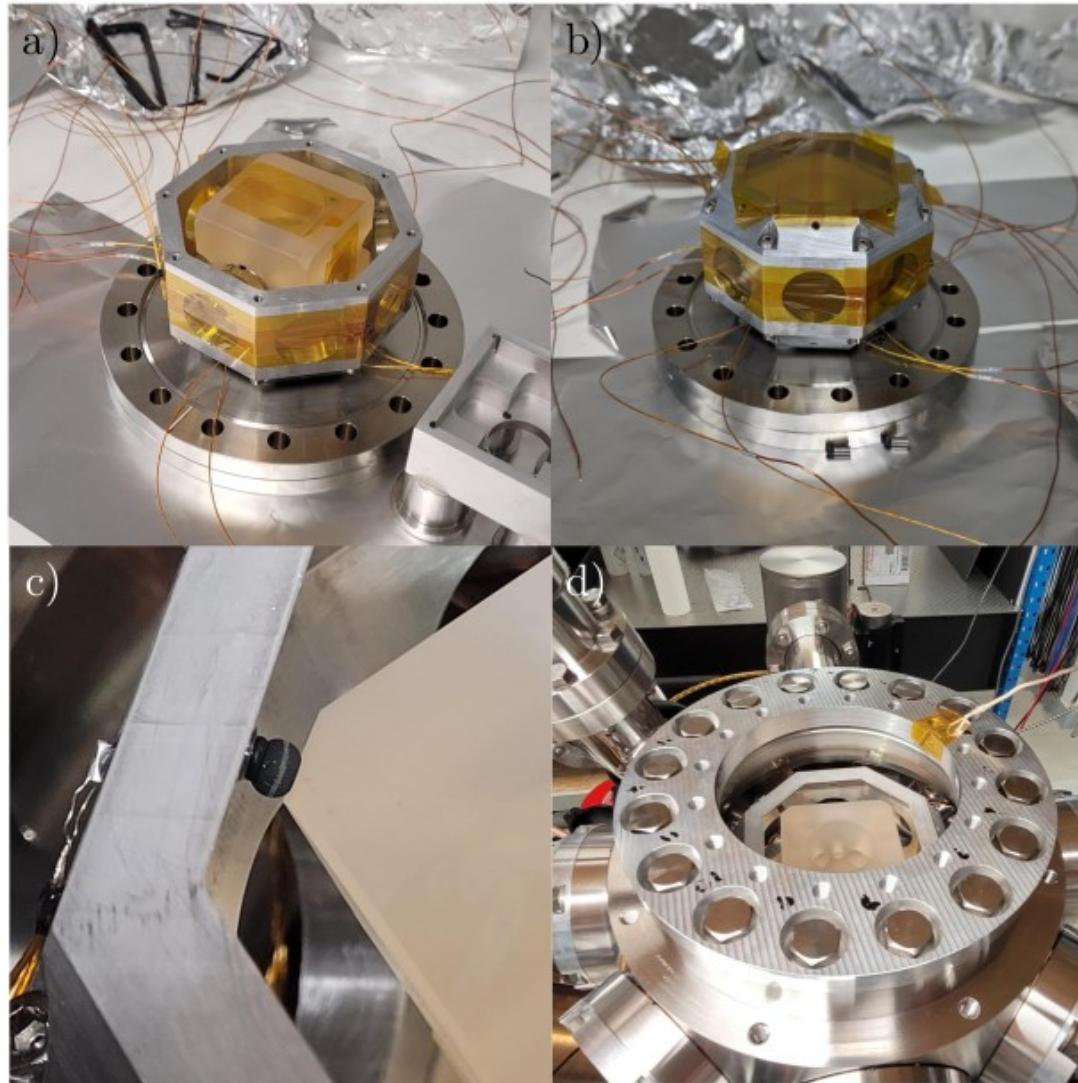
and then (Zeeman viewport) ...



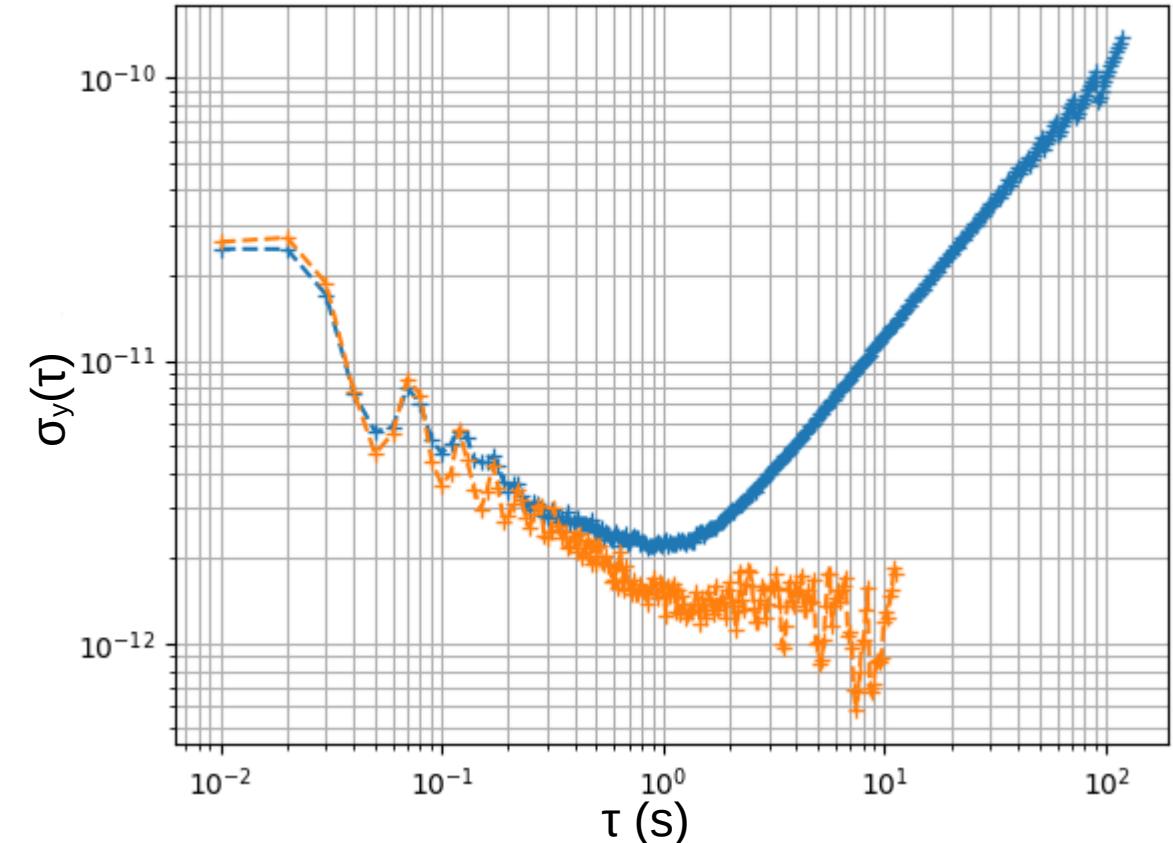
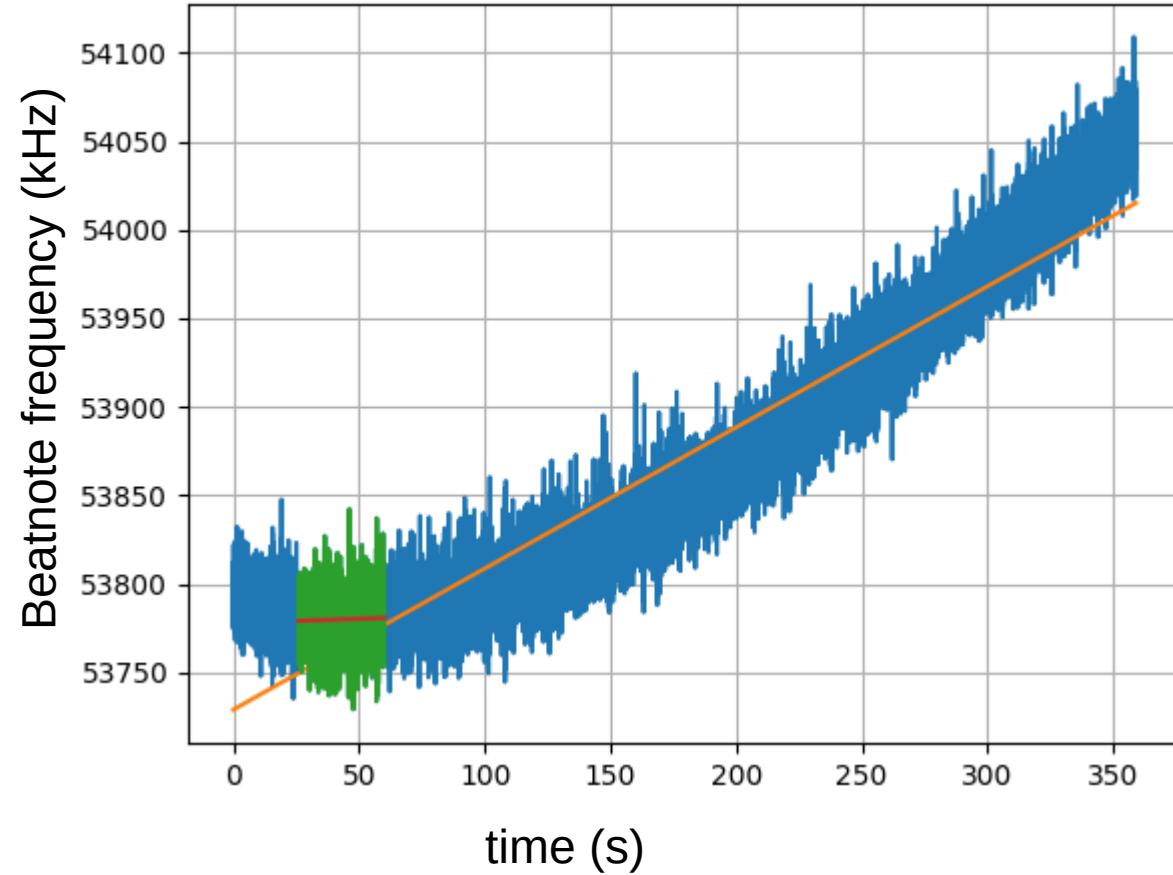
# Cavity assembly



# Cavity in vacuum



# Cavity characterization – short term



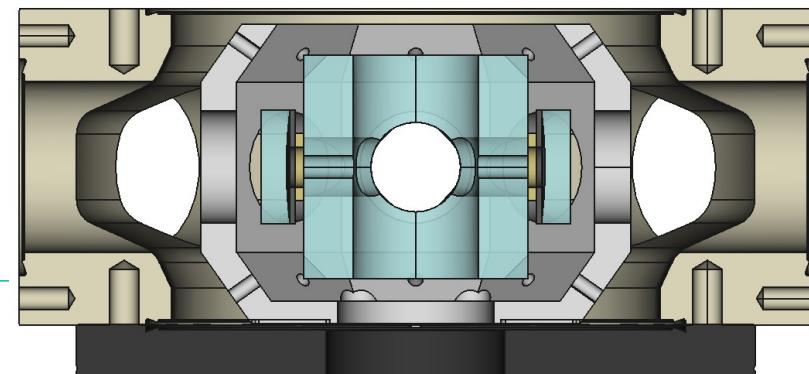
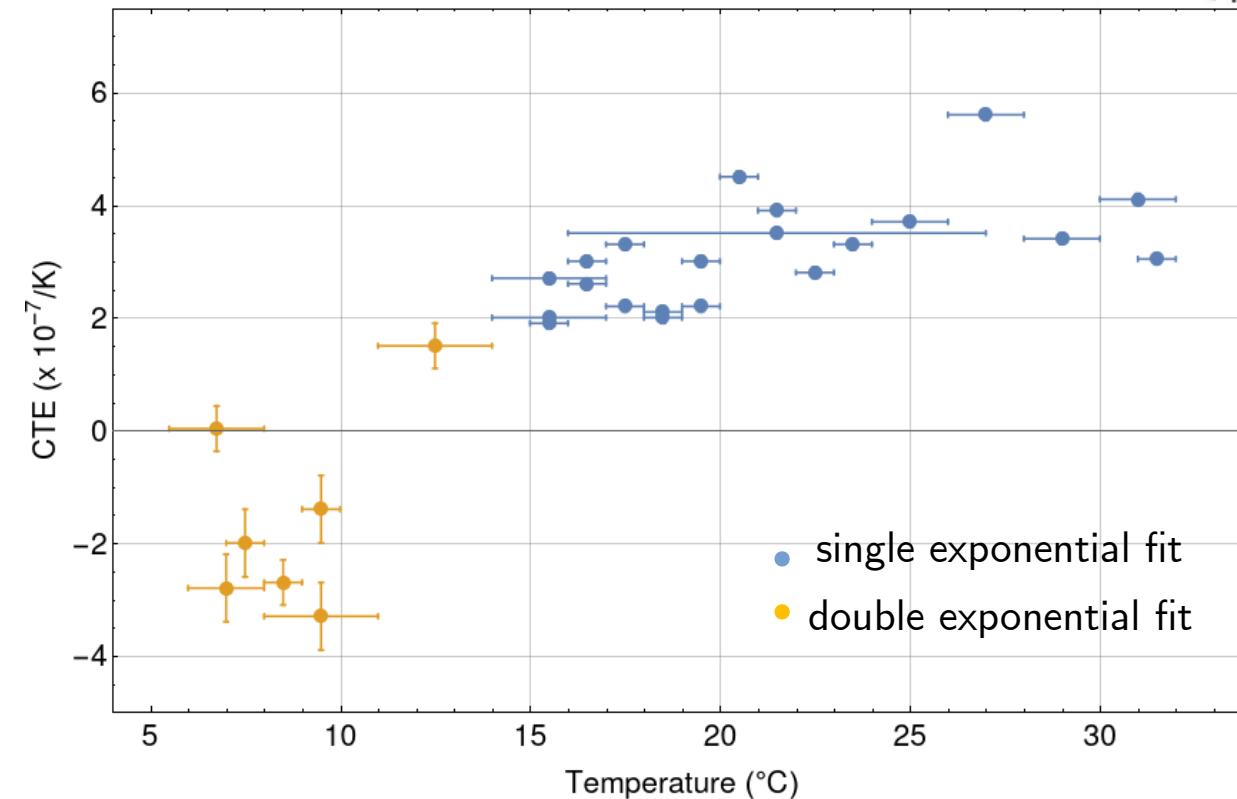
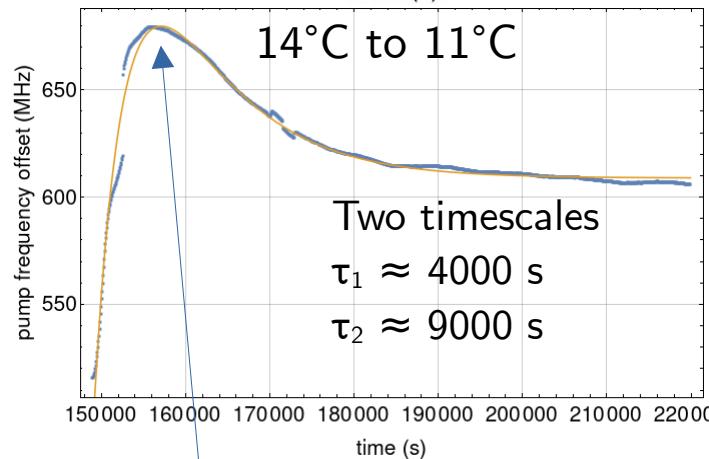
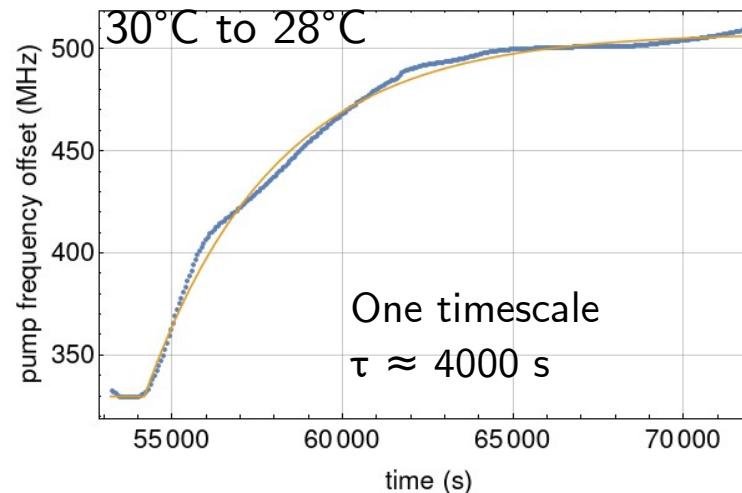
$$\sigma_y(\tau) = 2 \times 10^{-12} \text{ at } 1 \text{ s}$$

Work in progress: improve PDH laser lock on the cavity – limited by drifts after 1s

# Cavity characterization – long term

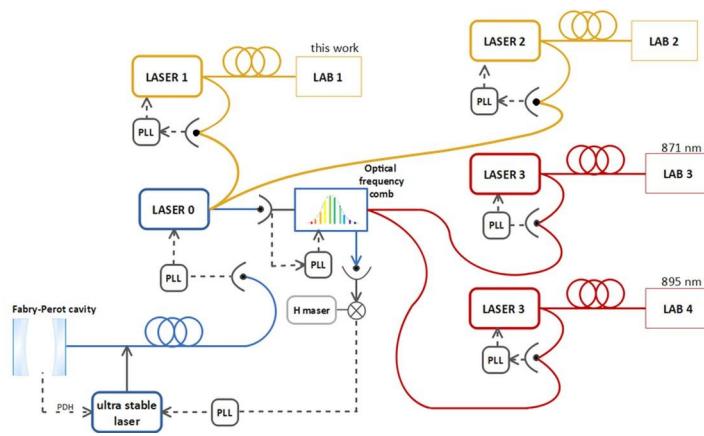


## Thermal expansion coefficient

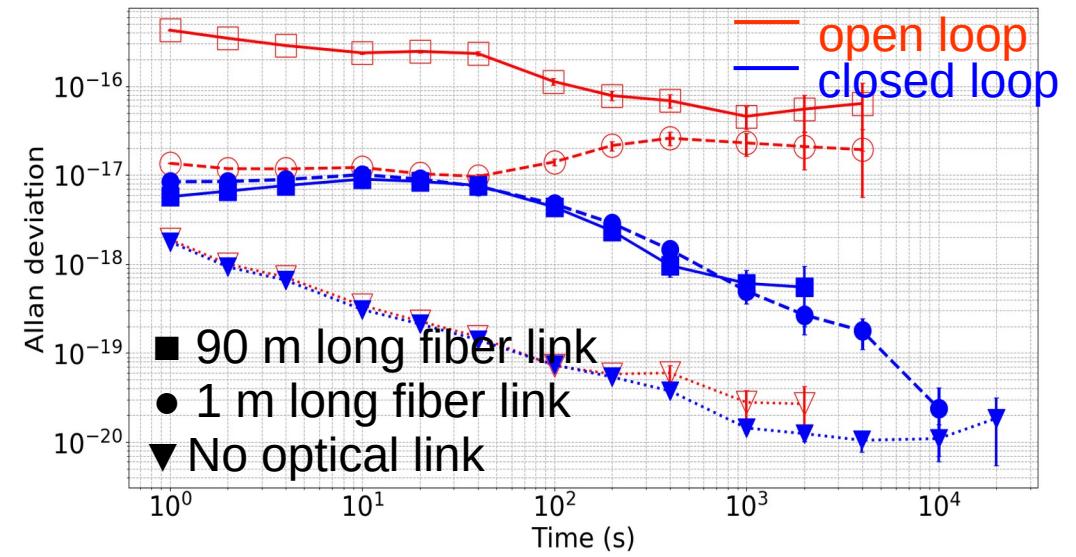
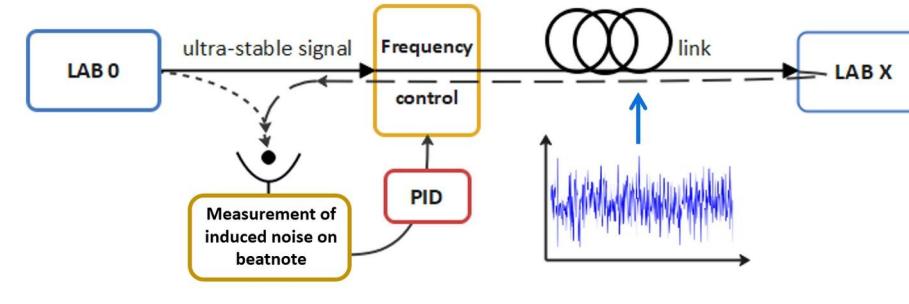
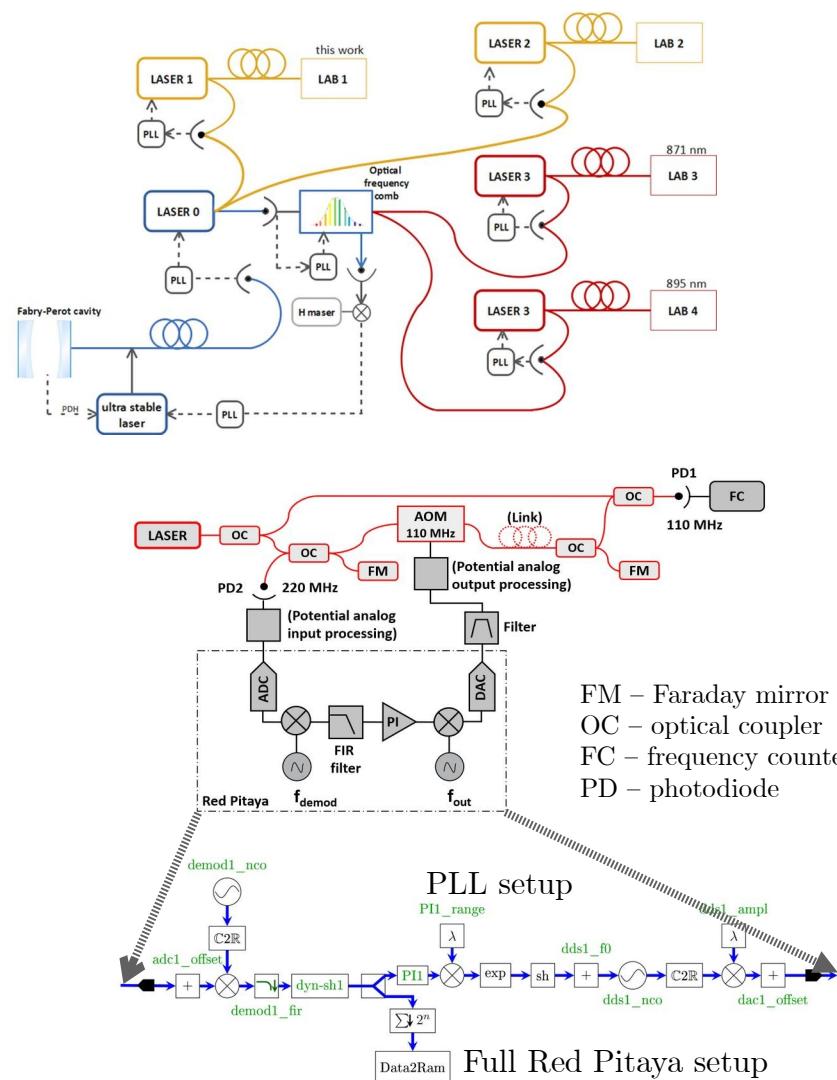


Possible crossing of inversion temperature <https://doi.org/10.1364/OE.436112>

# Optical fiber links



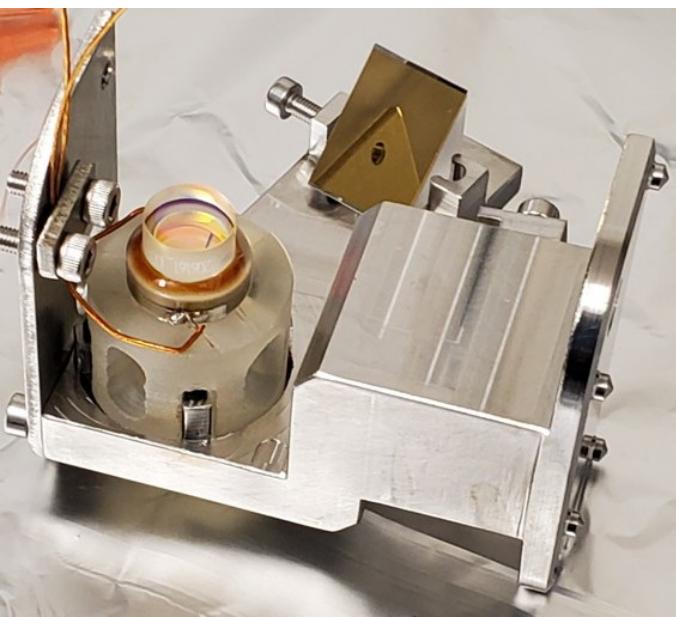
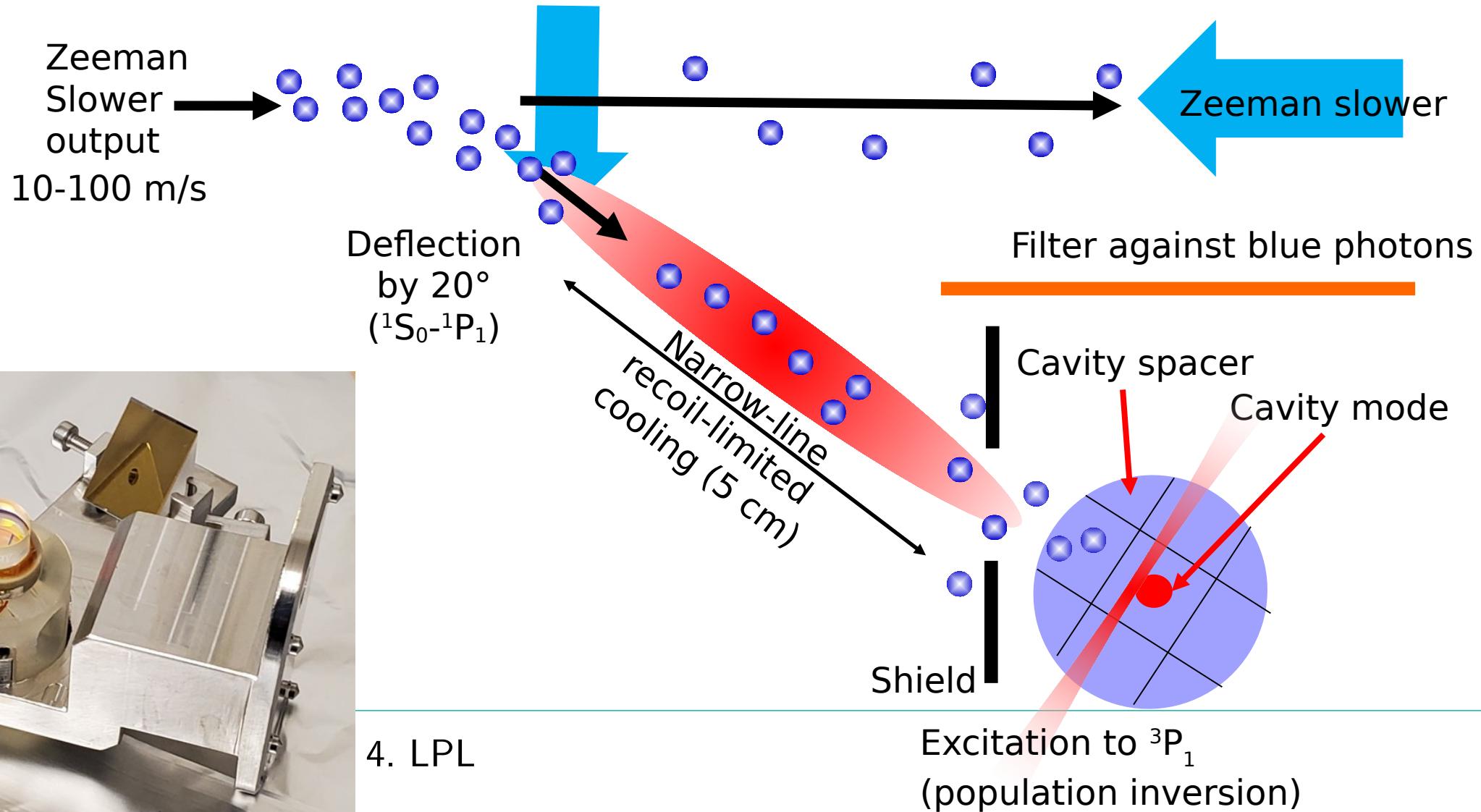
# Optical fiber links



→ OK for optical clock comparison at the  $10^{-18}$  level

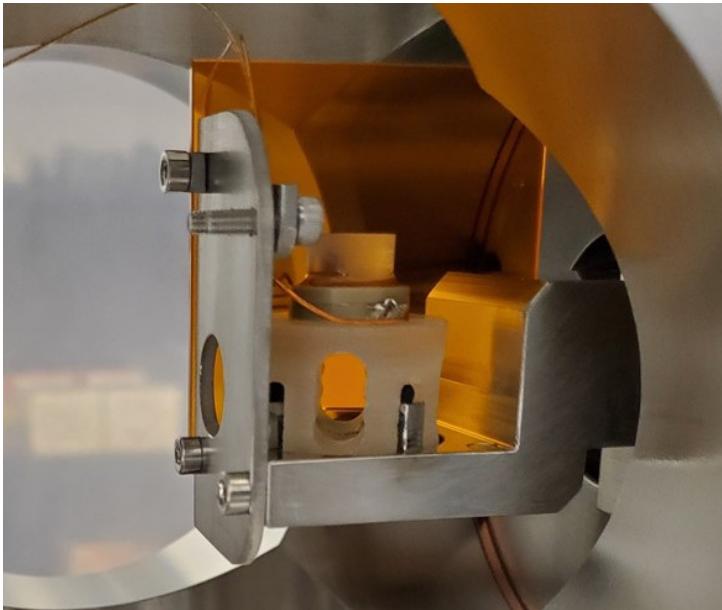
M.Matusko et al, Rev. Sci. Instrum. 94, 034716 (2023)

Strontium 88 superradiant laser on the 7 kHz wide intercombination line  $^1S_0 \rightarrow ^3P_1$



4. LPL

# The LPL atomic-beam architecture



*Finesse measured in vacuum : 9000 (over ISL 6.6 GHz)  
Mirror losses 0.028% reduce outcoupling by a factor ~ 10*

Mode waist :  $68 \mu\text{m}$

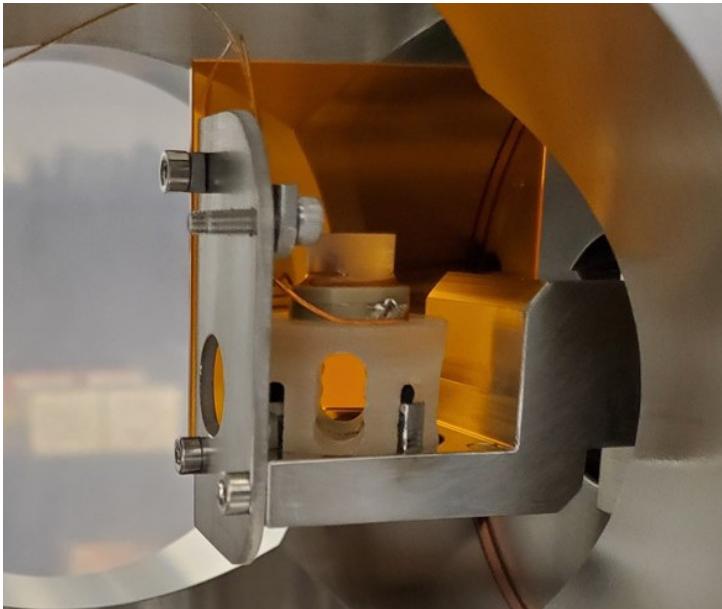
$g/2\pi = 20 \text{ kHz (rms)}$

$\gamma/2\pi = 7 \text{ kHz}$

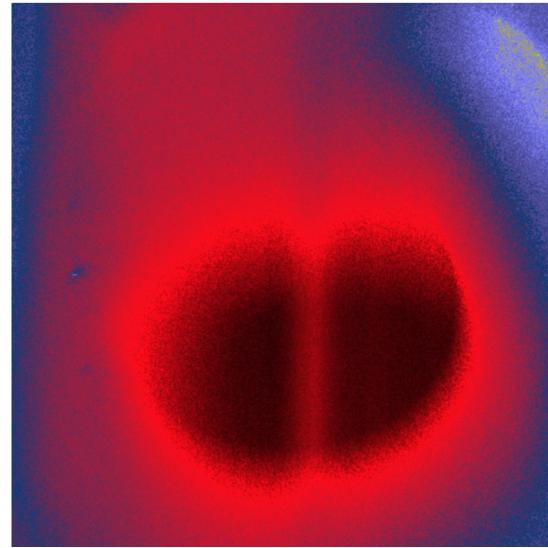
$\kappa/2\pi = 630 \text{ kHz}$

**C = 0.1** (spatial mean)

## The LPL atomic-beam architecture



Atomic beam decelerated and deflected by a moving optical molasses to the cavity



*Finesse measured in vacuum : 9000 (over ISL 6.6 GHz)  
Mirror losses 0.028% reduce outcoupling by a factor ~ 10*

Mode waist : 68  $\mu\text{m}$   
 $g/2\pi = 20 \text{ kHz (rms)}$   
 $\gamma/2\pi = 7 \text{ kHz}$   
 $\kappa/2\pi = 630 \text{ kHz}$   
**C = 0.1** (spatial mean)

**Tunable** transit time broadening (40-200 kHz) through atomic beam axial velocity : 20 – 100 m/s  
**Tunable** Doppler shift distribution (down to 100 kHz) through atomic beam transverse velocity spread (operation starting now)

Our experiment is designed around “high” cooperativity to reach threshold at low atomic flux :

Oven temperature  $440\text{ }^{\circ}\text{C} \rightarrow N \sim 10^3 \rightarrow$  power  $\sim 50\text{ pW}$

Although  $\gamma/2\pi \sim 7\text{ kHz}$ , each atom emits at rate  $\sim 100\text{ kHz}$ , and the linewidth is  $700\text{ Hz}$

For metrological application, reduce C (increase  $\kappa$ ) and use high atomic flux

- linewidth reduced  $g^2/\kappa$
- Cavity pulling resilience  $\sim 1/\kappa\tau$

## PHYSICAL REVIEW LETTERS

Rugged mHz-Linewidth Superradiant Laser Driven by a Hot Atomic Beam

Haonan Liu, Simon B. Jäger, Xianquan Yu, Steven Touzard, Athreya Shankar, Murray J. Holland, and Travis L. Nicholson

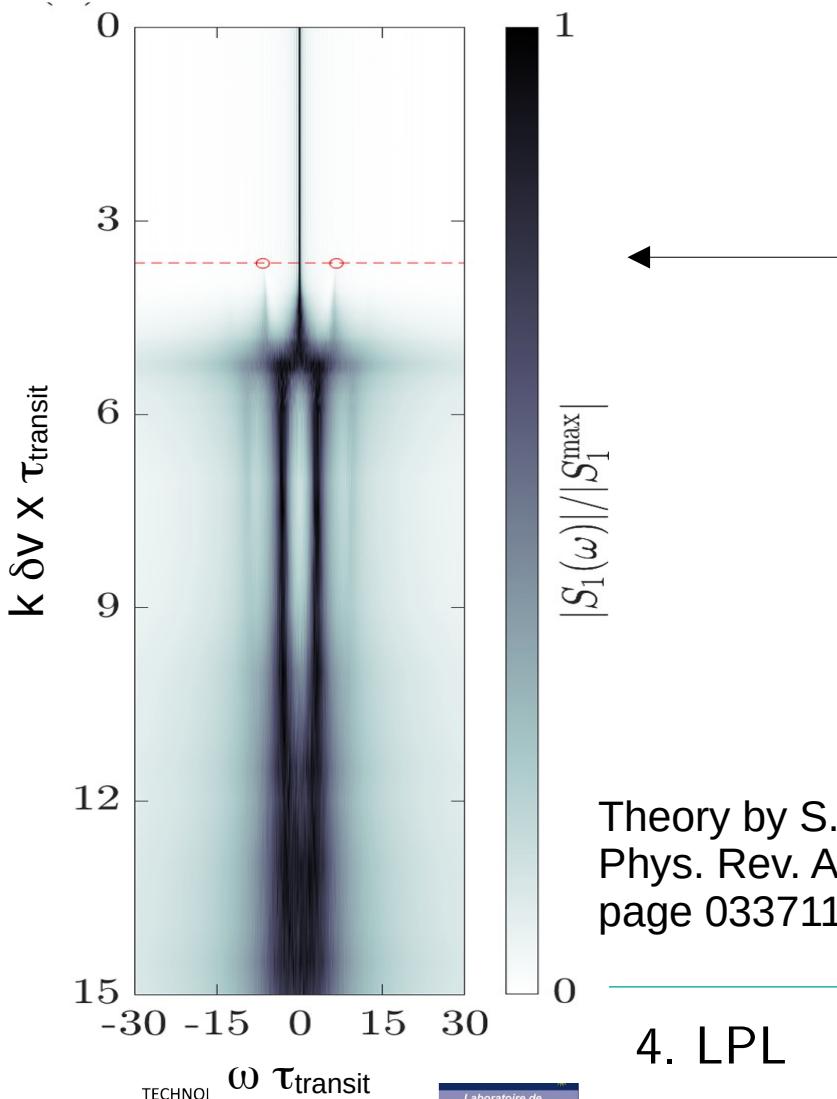
Phys. Rev. Lett. **125**, 253602 – Published 18 December 2020

$C = 2 \cdot 10^{-5} \rightarrow \delta\omega = 0.2\text{ Hz}$   
Cavity pulling rejection 0.004  
Power 2  $\mu\text{W}$   
(for  $650\text{ }^{\circ}\text{C}$  oven ... )

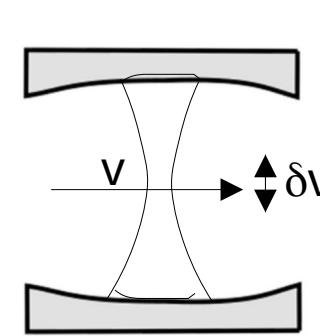
## LPL design choices : mode waist



Tunable transit time and Doppler shift distribution



For Doppler spread  $k \delta v = 200 \text{ kHz}$ ,  
with  $2 \times 10^9 \text{ atoms /s}$  through mode,  
our experiment at beam velocity  $50 \text{ m/s}$

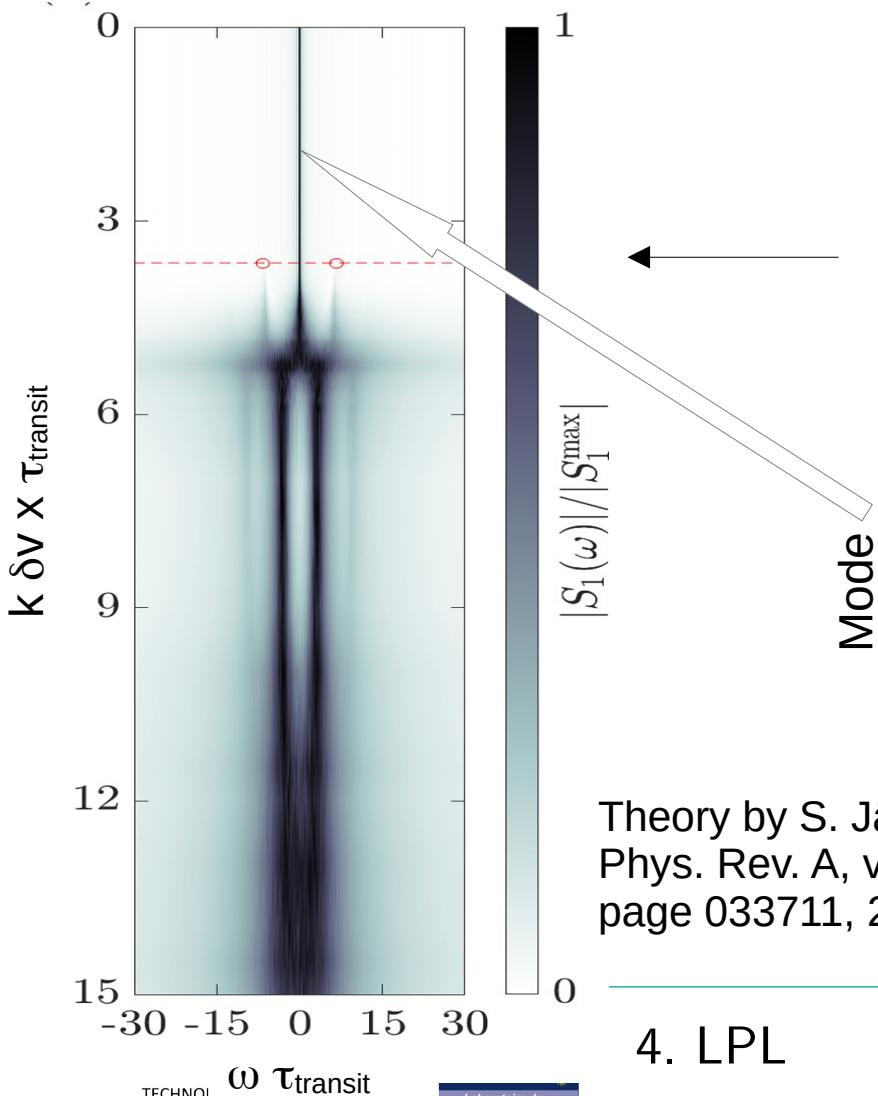


Theory by S. Jäger et al,  
Phys. Rev. A, vol. 104,  
page 033711, 2021

# LPL design choices : mode waist

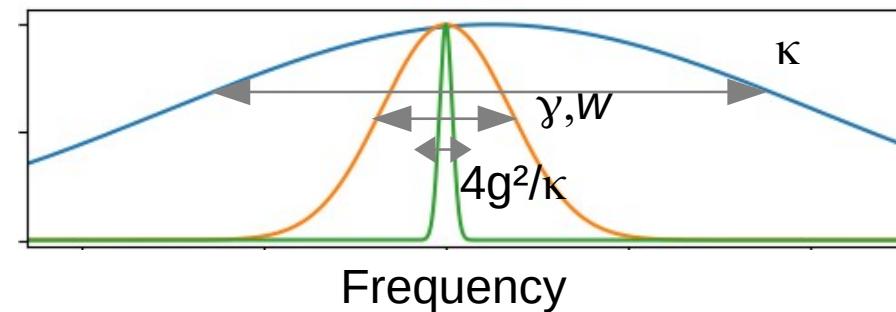


## Tunable transit time and Doppler shift distribution

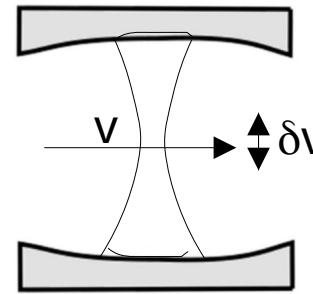


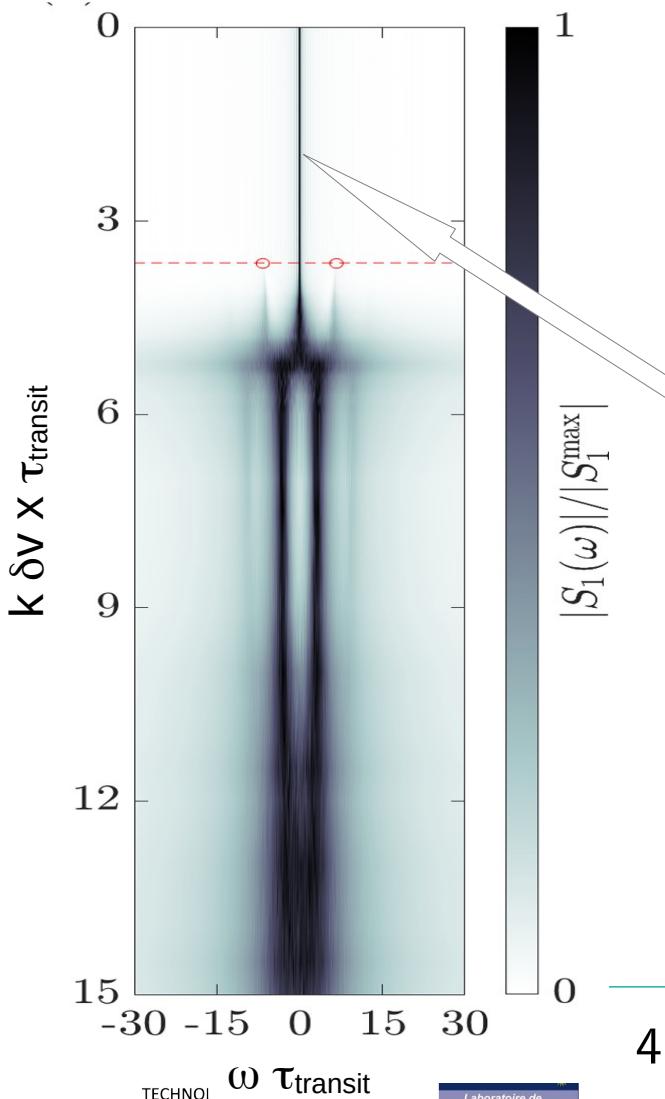
For Doppler spread  $k \delta v = 200$  kHz,  
with  $2 \times 10^9$  atoms /s through mode,  
our experiment at beam velocity 50 m/s

Monomode, sub-natural-linewidth



For us,  $\kappa \sim 630$  kHz,  
 $w \sim 100$  kHz,  
 $\gamma = 7$  kHz  
 $4g^2/\kappa \sim 700$  Hz





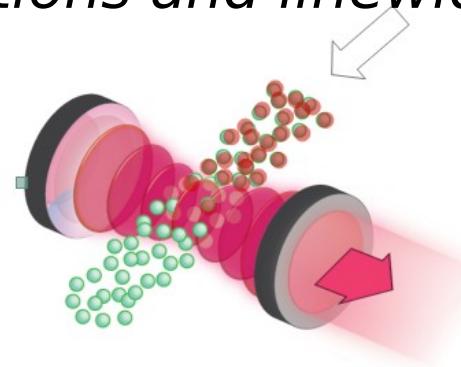
**Theoretical literature of the atomic-beam SR laser based on Monte Carlo approaches**

**Our contribution :**

- a **stand-alone** theoretical paper (from basics)
- **analytical expression** for threshold, power, atom-atom and atom-field correlations
- a picture to explain the **linewidth  $< \gamma$**

Restriction : no Doppler shift, resonant cavity

→ Laburthe-Tolra et al, Scipost Physics Core 6, 015 (2023)  
*Correlations and linewidth of the atomic beam superradiant laser*



We derive the equations of evolution for:

Intra-cavity atom population inversion ( $s_z$ )

Intra-cavity atom dipole quadrature ( $s_x, s_y$  or  $s_+, s_-$ )

Intra-cavity field ( $b, b^+$ )

**Infinite set of coupled equations**  
(cumulant expansion)

cf Debnath, Zhang and Molmer, PRA 2018

$$\frac{d \langle s_j^- \rangle}{dt} = 2ig \langle s_j^z b \rangle - \frac{\Gamma}{N} \langle s_j^- \rangle - \frac{\gamma}{2} \langle s_j^- \rangle$$

$$\frac{d \langle s_1^z b \rangle}{dt} = \dots$$

$$\langle AB \rangle = \langle A \rangle \langle B \rangle$$

$$\langle ABC \rangle = \langle AB \rangle \langle C \rangle + \langle BC \rangle \langle A \rangle + \langle CA \rangle \langle B \rangle - 2 \langle A \rangle \langle B \rangle \langle C \rangle$$

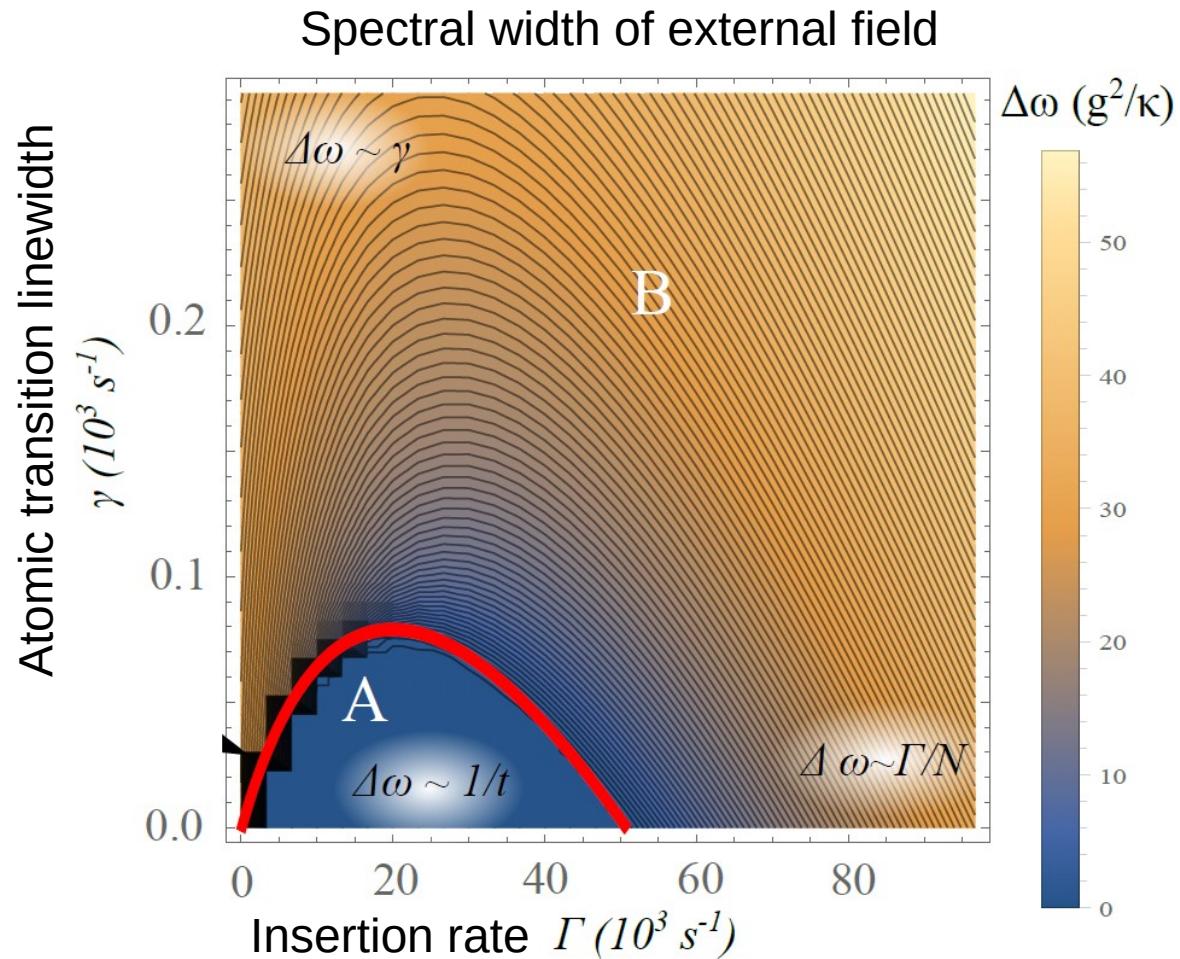
Choose  
approximation  
degree

Mean-field :

To second order :

# Within mean field approximation ( $\langle A \cdot B \rangle = \langle A \rangle \langle B \rangle$ ) : understand the threshold

**Plot at  
 $N$  constant**



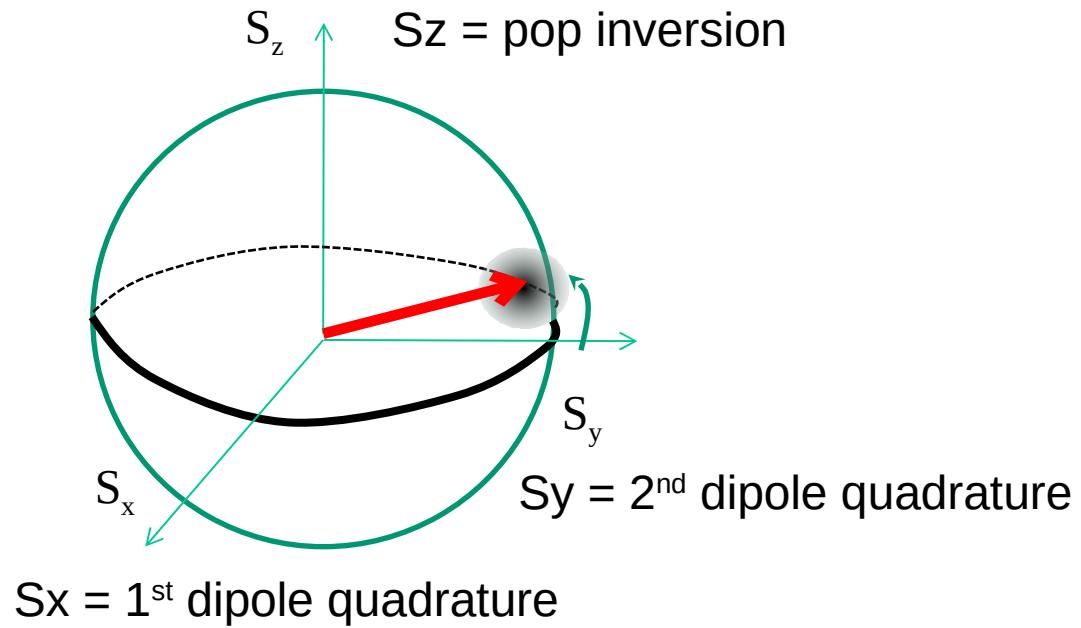
$\Gamma$ : insertion rate:  
 $\Gamma_R = \Gamma/N$ : transit rate (Refreshing rate)

Red line : **analytical** requirement for  
steady-state macroscopic dipole

implies

$$\left. \begin{aligned} \Gamma_R &> \gamma \\ N \frac{g^2}{\kappa} &= NC \gamma > \frac{\Gamma_R}{2} \end{aligned} \right\}$$

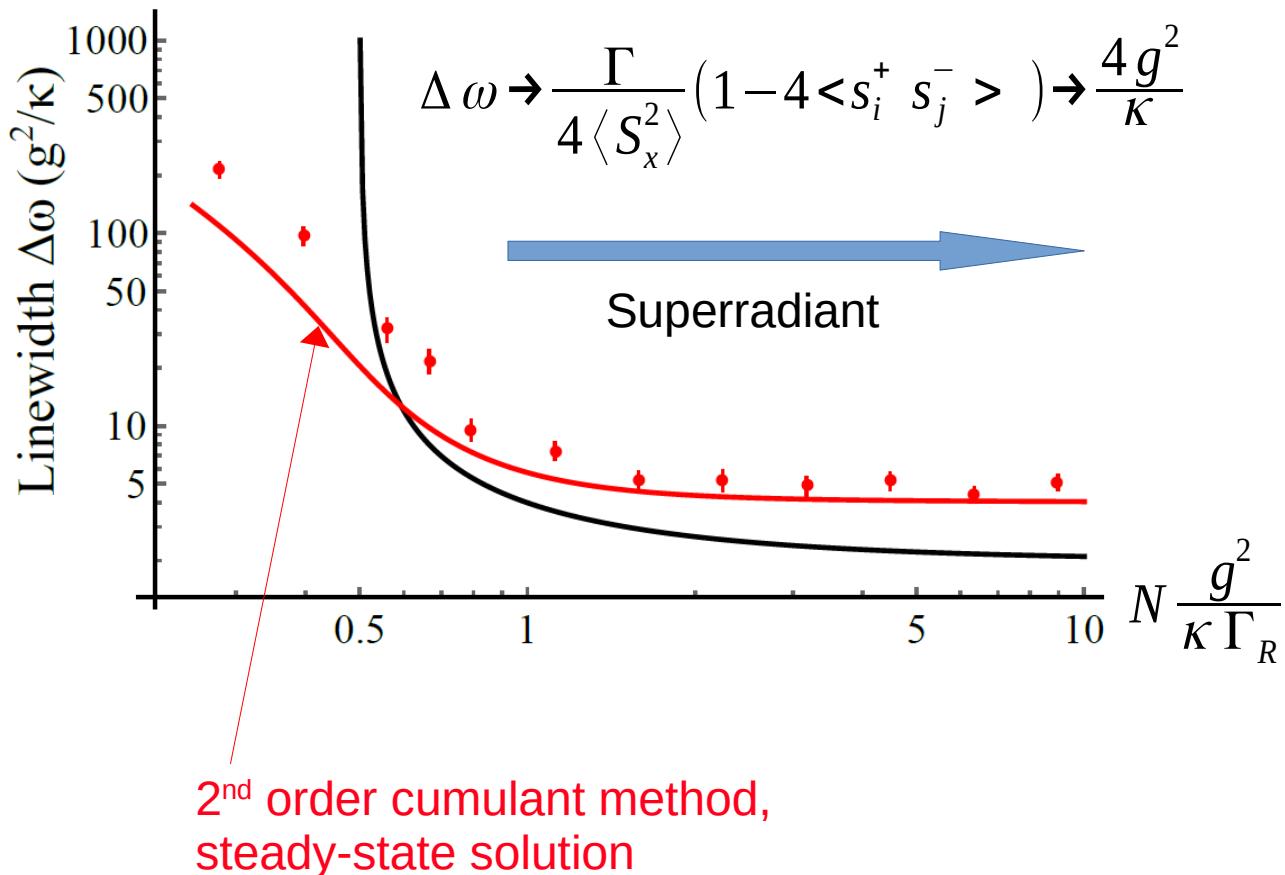
Reminder : the electric field follows the collective atomic dipole



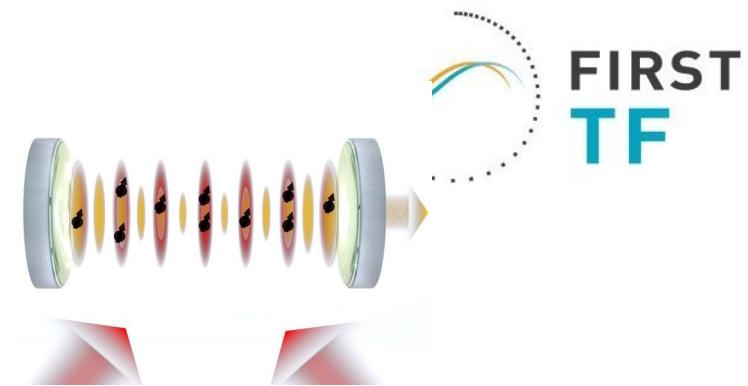
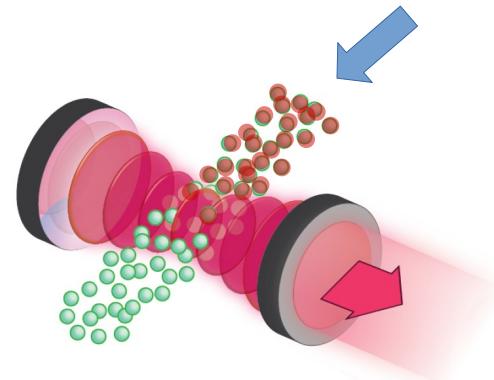
Collective dipole phase diffusion:

$$\langle S_x(t + dt)S_x(t) \rangle = \langle S_x^2 \rangle \exp\left(-\frac{dt}{\tau_c}\right)$$

$$\boxed{\frac{1}{\tau_c} = \Delta\omega}$$



## Developing the two architectures



Complexity

Rather low

Rather high (trapping, cooling, conveyor belt, repumping lasers ...)

Optical transitions

The threshold requirement  
 $NC\gamma > 1/t_{\text{transit}} > k \delta v$   
requires more atoms than  $NC > 1$   
→ **require “broad” line**  
 $\gamma \sim \text{optical recoil energy optimal}$

Motion essentially frozen  
Main condition remains  $NC > 1$   
→ **Clock-line compatible**

Shifts

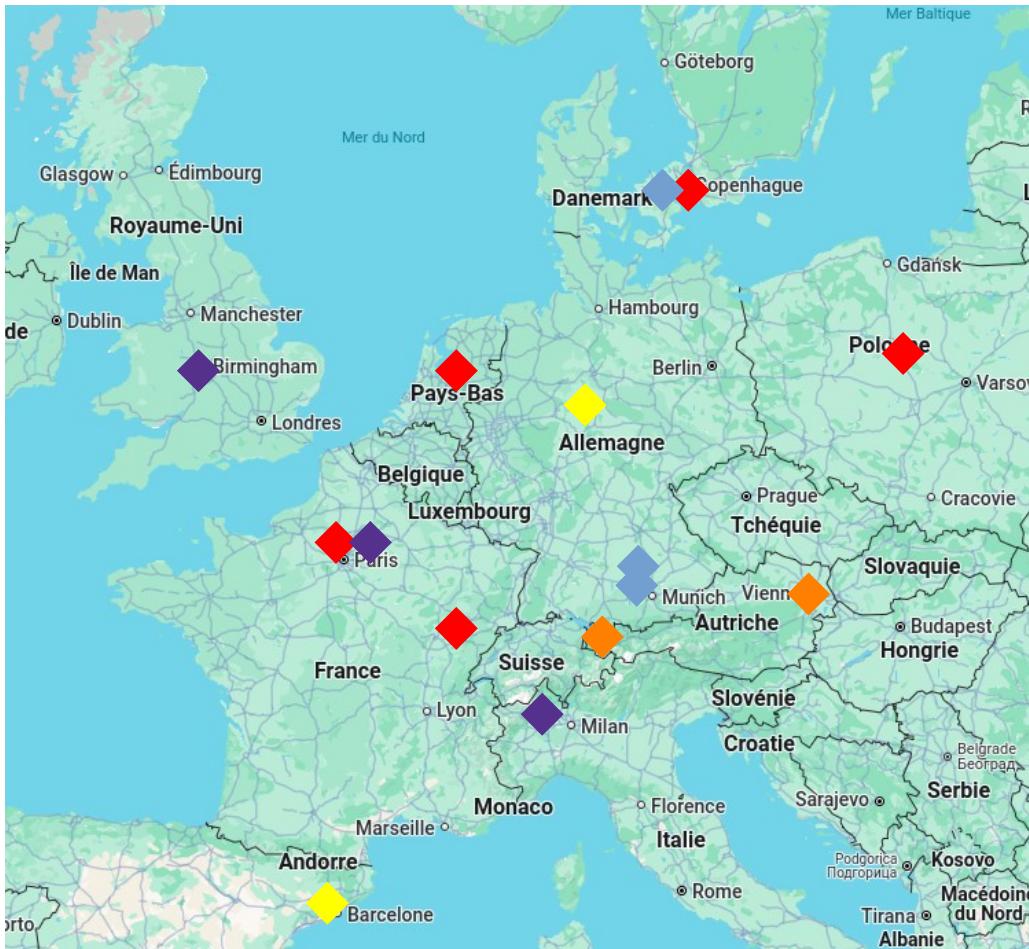
No first order transverse Doppler shift  
***if cavity symmetric***  
Second order Doppler shift at 20 m/s : - 1 Hz

Requires “magic” trapping wavelength  
Many laser fields (repumping, cav lock)  
Complex vacuum system (black body)

Role

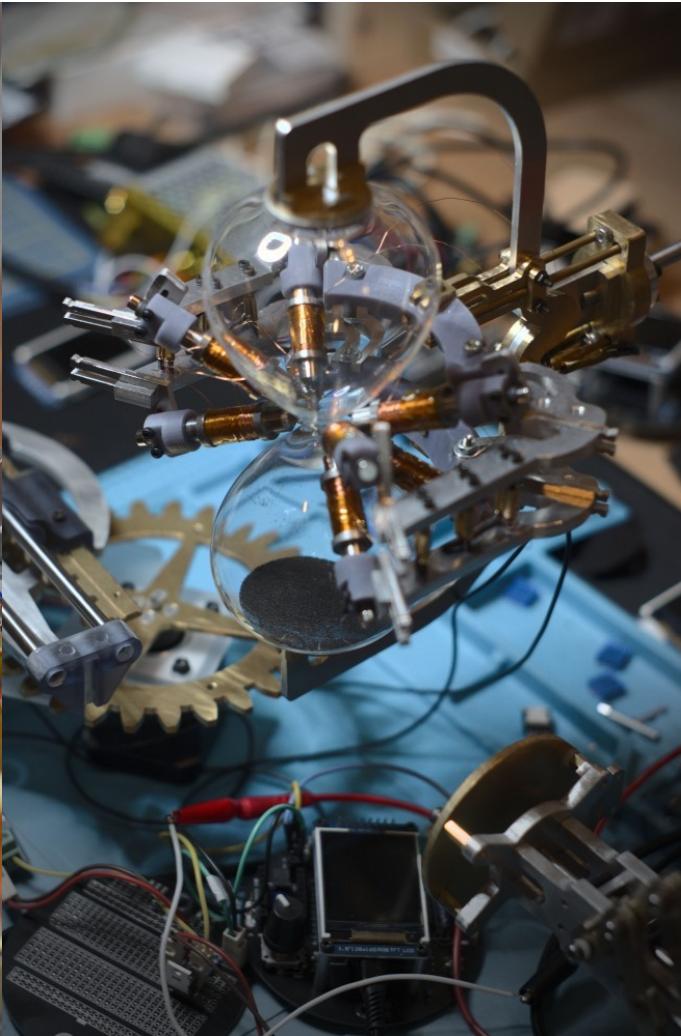
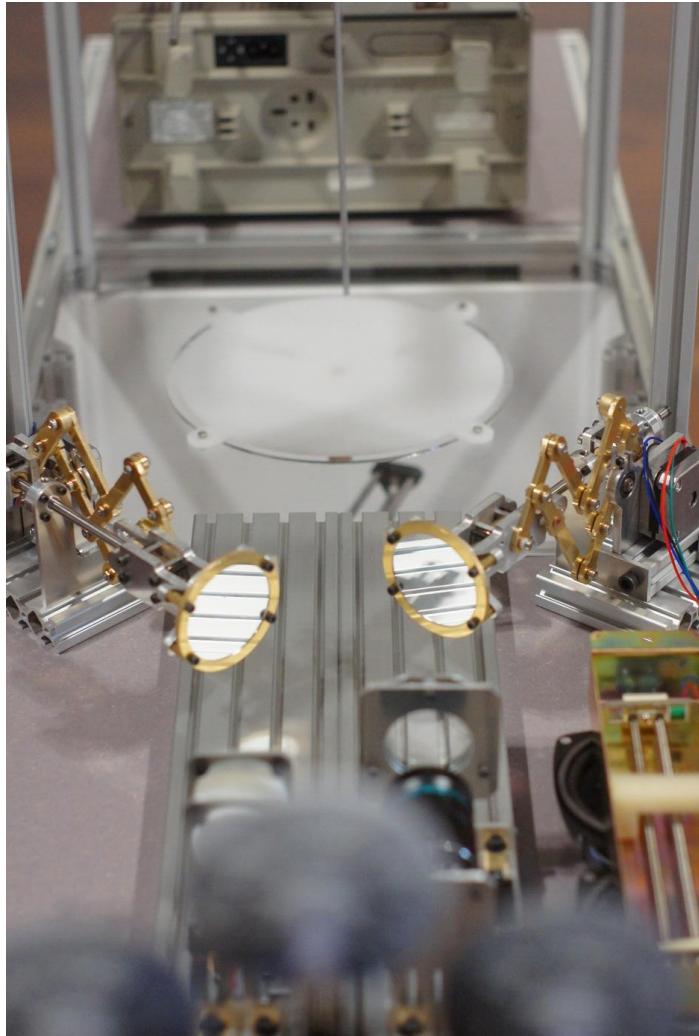
Optical-domain flywheel

From optical-domain flywheel  
to “competitor” to lattice clocks?  
→ **investigations required on systematic effects**



## QURIOUS proposal (ITN)

- ◆ Superradiant laser (exp.)
- ◆ Superradiant laser (th.)
- ◆ QND, squeezing
- ◆ Industrial partner
- ◆ Other academic partner



Funded by ANR  
CONSULA project