Flicker noise in ultra-stable bulk acoustic wave resonators: characterization and modeling

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Besançon

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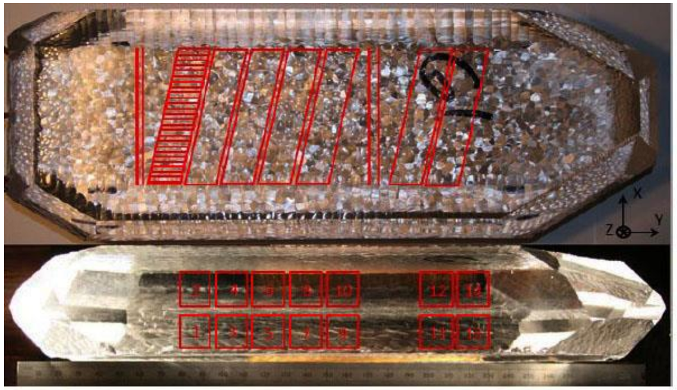
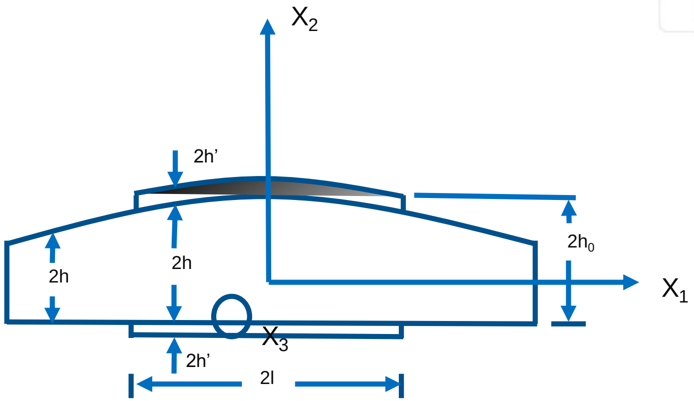
Ultra-stable quartz crystal resonator

Features:

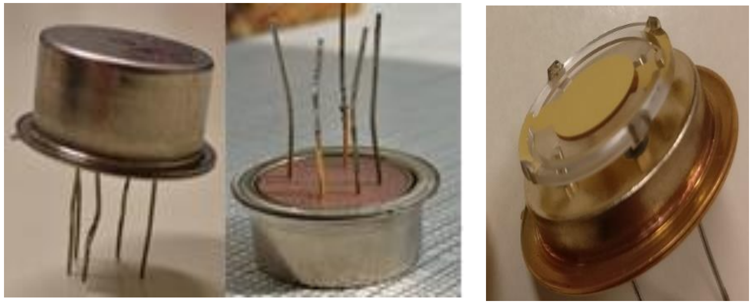
- 5 MHz- Bulk Acoustic Wave (BAW) - doubly-rotated SC-cut- Quartz Plano-convex resonator plate.



Quartz



Tiersten's plano-convex model.

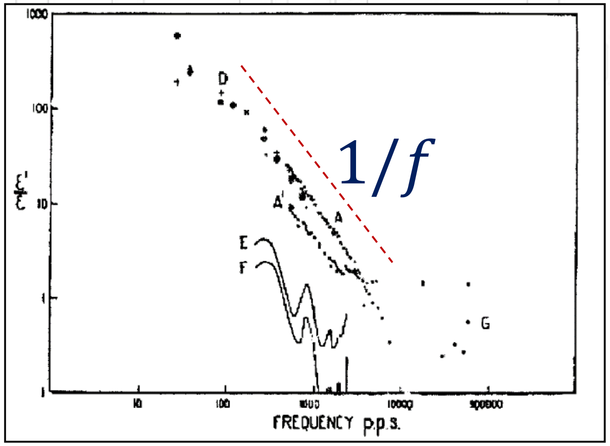


Quartz plate mounted resonator

F. Sthal, M. Devel, J. Imbaud, R. Bourquin, S. Ghosh, G. Cibiel, "Study on the origin of 1/*f* noise in quartz resonators," J. Stat. Mech., n° 6, May, 054025, 2016.

Brief history of 1/f noise (Flicker Noise)

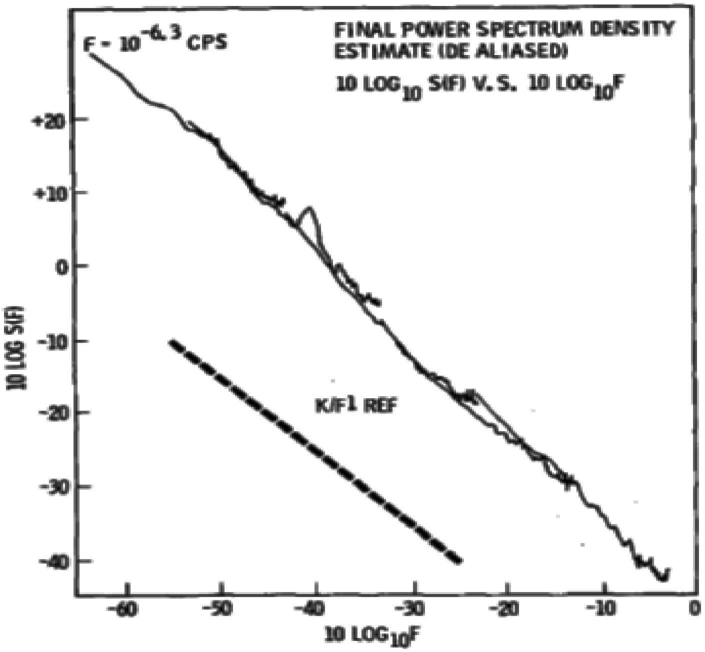
First observations: Johnson discovered **Flicker noise** accidentally in vacuum tubes whose PSD was equivalent to $1/f$



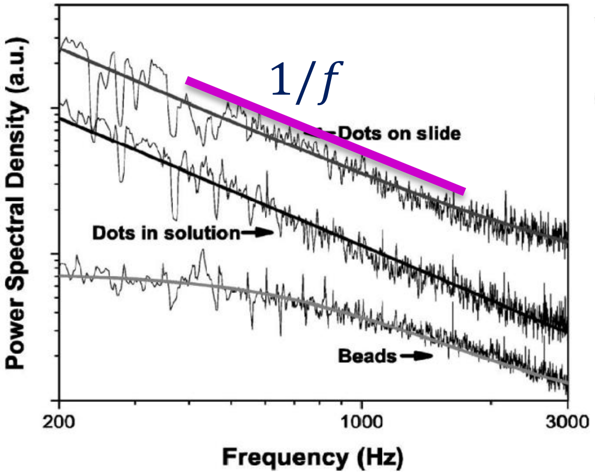
A decade later $1/f$ observed in granular graphites *Christensen et al.* Bell Sys.Tech. J. 15 (1936) 197–223 and in thin films by *Bernamont,* Proc. Phys. Soc. (London) 49, (1937) 138–139

On mid 70's , *Caloyannides,* J. Appl. Phys., 45 (1974) 323

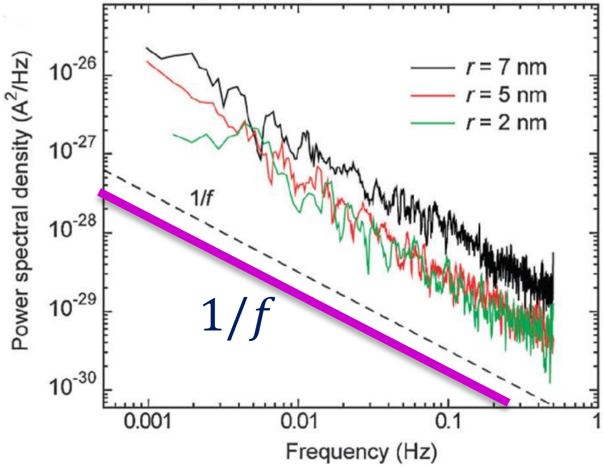
- Measured noise on Op-Amps.
- 3 weeks of measurement on millions of samples up to $10^{-6}\ Hz$ on Fourier frequency.
- No low frequency cutoff.
- Slope $1/f^{1.23}$



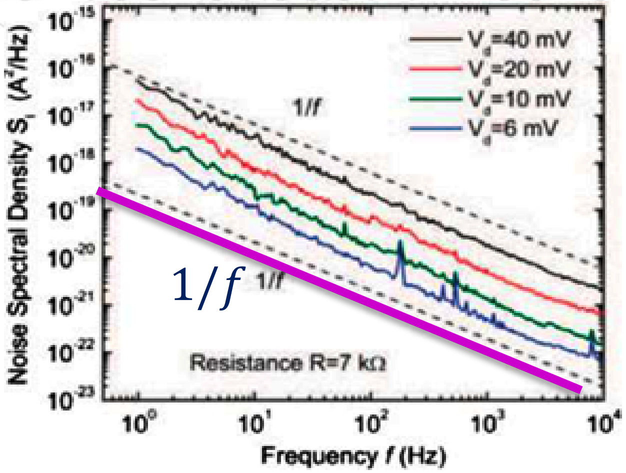
Brief history of 1/f noise (Flicker Noise)



Semiconductor nano-crystals
Pelton et al. Proc. Natl. Acad. Sci. U.S.A. 104, 14 249, 2007

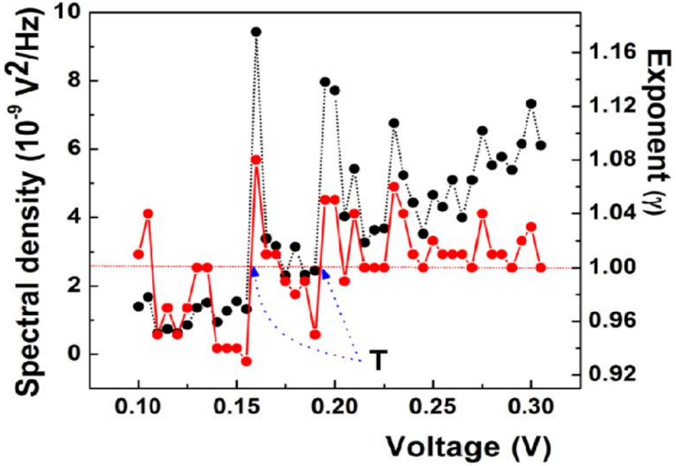


Nano-electrodes, *Krapf,* Phys. Chem. Chem. Phys. 15, 459, (2013)



van der Waals materials, *Karnatak et al. Adv. Phys.: X* 2, 428–449 (2017).

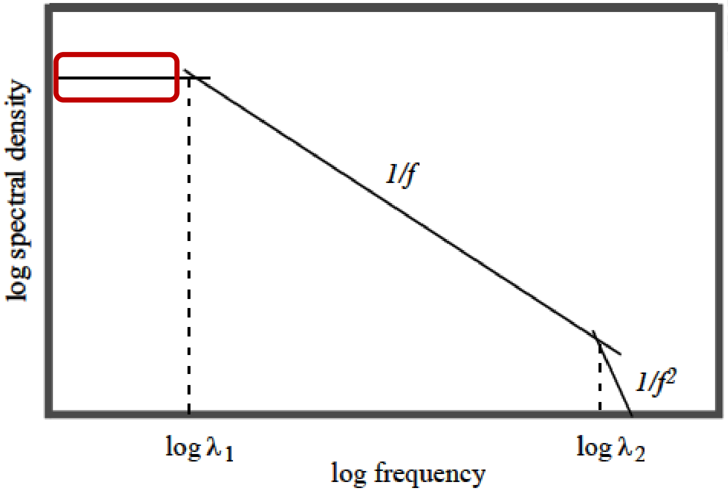
Most recent prediction of microscopic source of $1/f$ noise



- *Mihaela et al*, Nature Scientific Reports 9, (2019) 947 **1/f noise in carbon soot resistor**
- <u>Non-linearity</u> and <u>dispersion</u> causes transitions in the system.
- Source of non-linearity lies on **electron-phonon coupling**.

Brief history of 1/f noise (Flicker Noise)

- **Schottky (1926), Du Pré (1950) and Dutta et al (1979):**
  - Schottky:Superposition of power spectrums of fluctuations $S(f)$ are **lorentzians** = Du Pré: exponential with random relaxation time is given by
    $$\tau = \tau_0 \exp(E_A/k_B T)$$
    and a distribution of activation energies E_A almost uniform on a range of order $k_B T$, for a given (low) frequency interval
  - Dutta:Gives $1/f^\alpha$ noise, with $\alpha \approx 1$, in that given frequency interval
  - Pb: distribution of activation energies Q and origin of lower frequency unknown (so far no lower frequency ever measured…)



Fourier domain:

$$S(\omega) \approx \begin{cases} \dfrac{N_0^2 n}{\lambda_1 \lambda_2} & 0 < \omega \ll \lambda_1 \ll \lambda_2 \\ \dfrac{N_0^2 n}{2\omega(\lambda_2 - \lambda_1)} & \lambda_1 \ll \omega \ll \lambda_2 \\ \dfrac{N_0^2 n}{\omega^2} & \lambda_1 \ll \lambda_2 \ll \omega \end{cases}$$

λ is the relaxation rate

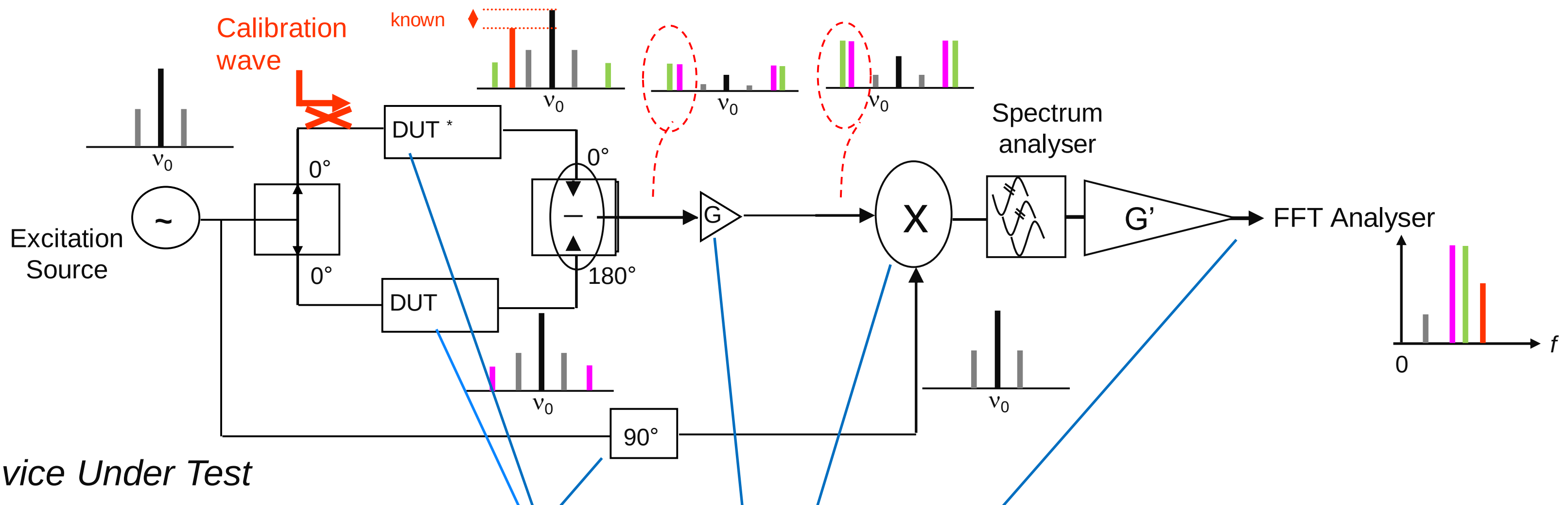
Recent Model (Niemann et al., 2013):

- Power law intermittency at low frequencies.
- Lower frequency ~ inverse of measurement time.
- Proposed two tests on adequation of their model.

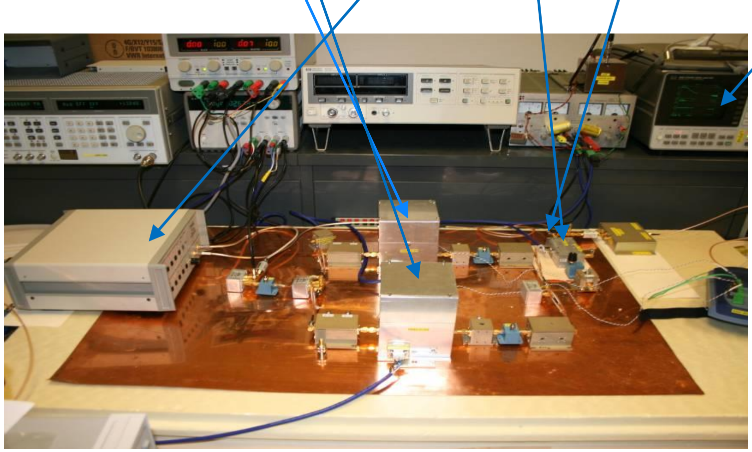
M. Niemann, H. Kantz, and E. Barkai,
“Fluctuations of 1/f noise and the low-frequency cutoff paradox,”
Phys. Rev. Lett., vol. 110, no. 14, p. 140603, 2013.

Phase noise measurement bench

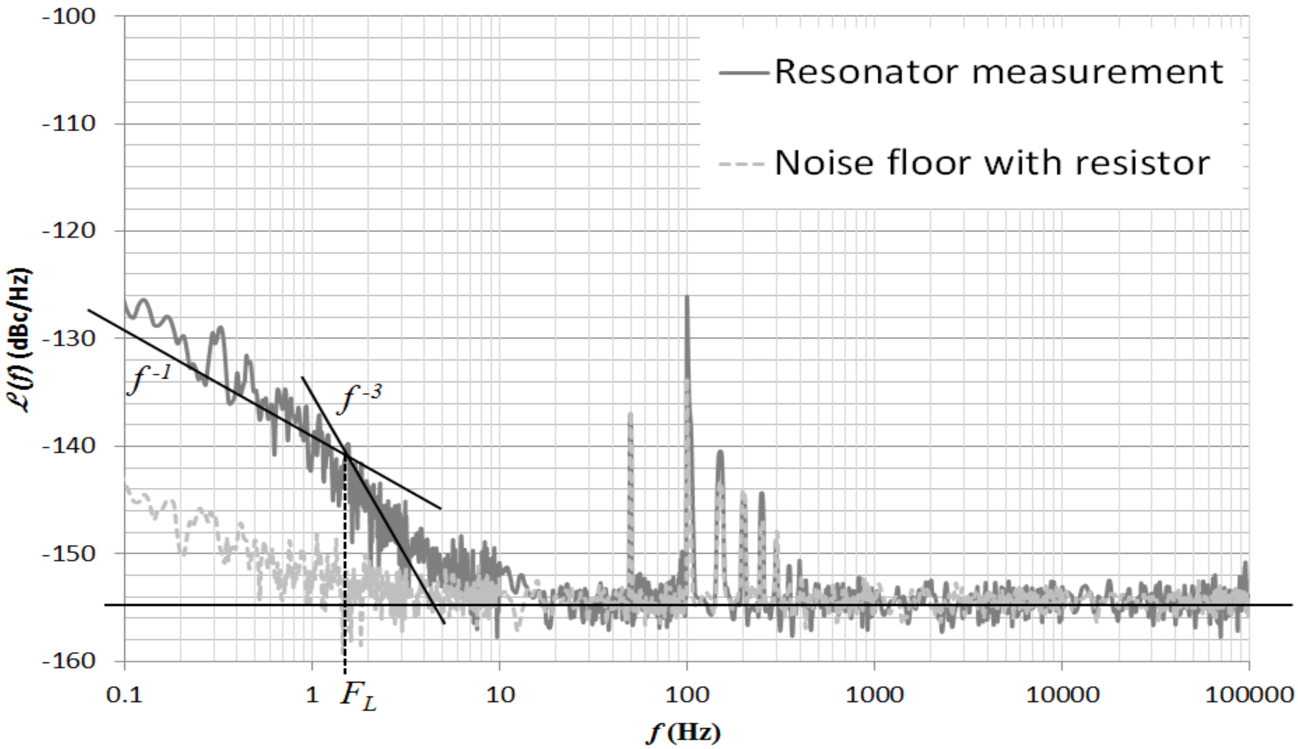
Carrier suppression passive bench



** DUT : Device Under Test*



Phase noise measurement results and obsv.



$$Q_L = \frac{f_{res}}{2 \cdot F_L}$$

- $f_{res.}$ = 5 MHz
- P_{xtal} = 55 µW
- F_L ≈ 2 Hz
- Q_L ≈ $1.25 \cdot 10^6$

$$\sigma_y = 5.2 \cdot 10^{-14}$$

ℒ(1 Hz) = -139 dBc/Hz

Allan standard deviation:

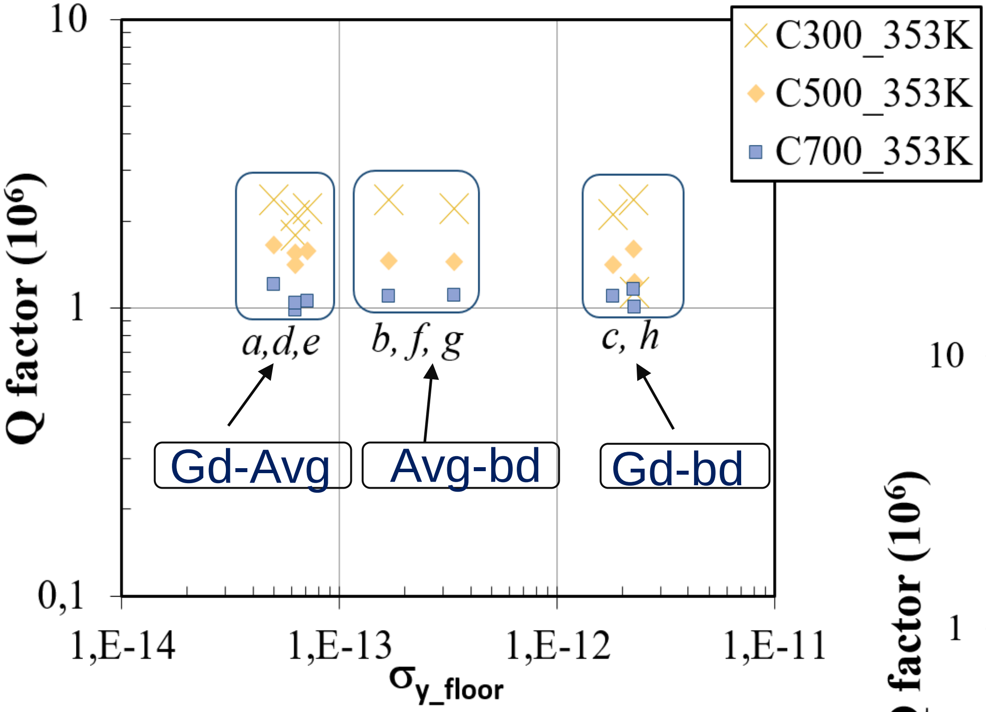
$$\sigma_{y\_floor} = \sqrt{2\ln(2) S_y(1Hz)}$$

- Power spectral density, $S_y(1\,Hz) = \left[\frac{F_L^2+1}{f_{res}^2}\right] S_\varphi(1\,Hz)$ & ℒ(1 Hz) = $S_\phi(1Hz)$(**identical resonator pairs**)

Q_L is the loaded quality factor, ℒ(f) Single sideband PSD of phase fluctuations

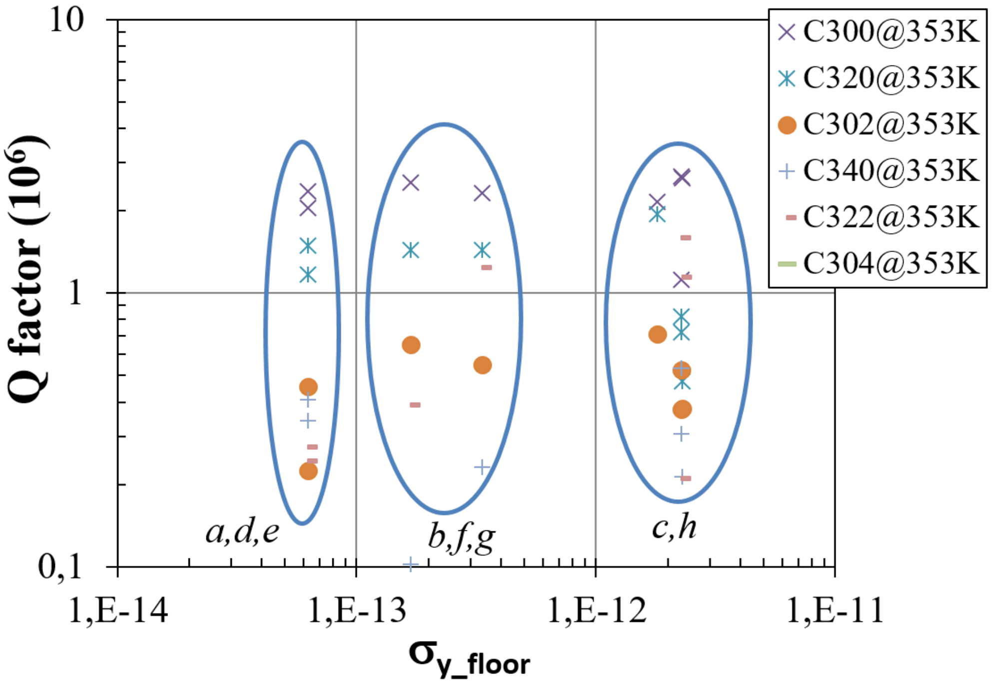
Experimental results at 353 K

Harmonic modes of resonance



- ***8 Resonators named from a to h****
- ***Gd-Avg – Good -average ($\leq 10^{-14}$ to $\leq 10^{-13}$)***
- ***Gd-bad – Good -bad ($\leq 10^{-14}$ to $\geq 10^{-12}$)***
- ***Avg-bad – Average -bad ($\cong 10^{-13}$ to $\geq 10^{-12}$)***

Anharmonic modes of resonance



Results:

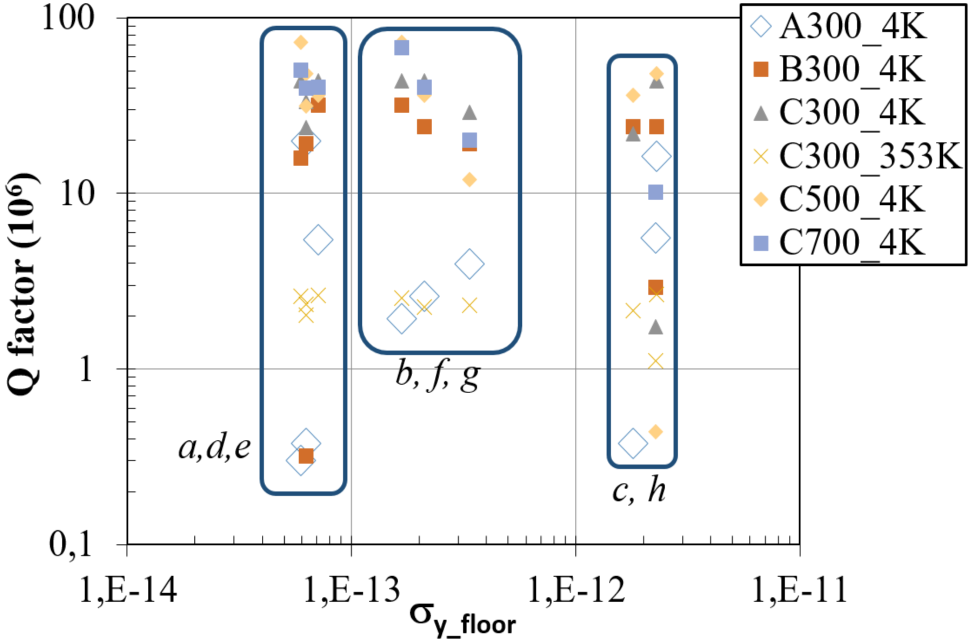
- ➢ No significant relation seen between Q-factor and Phase noise- both in harmonic and anharmonic modes.

***A. Pokharel**, F. Sthal, J. Imbaud, M. Devel, F. X. Esnault, G. Cibiel, "Flicker noise in quartz crystal resonator at 353 K as a function of Q factor of overtones and anharmonic modes at 4 K", Fluct. and Noise Lett, Vol. 17, n˚ 2, 1871002, 2018.

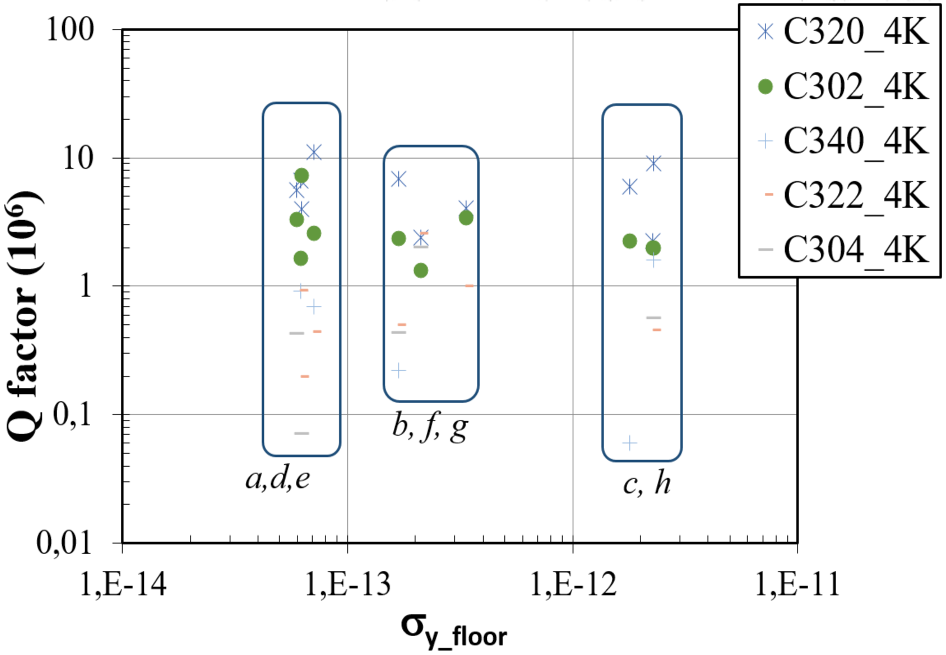
Experiments performed at 4 K

- Same kind of phase noise measurements technic as in 353 K.
- Two staged pulse tubed cryo cooler used.
- Extreme increase in Q-factor observed both in hamonic and anharmonic modes.

<u>Harmonic modes of resonance</u>



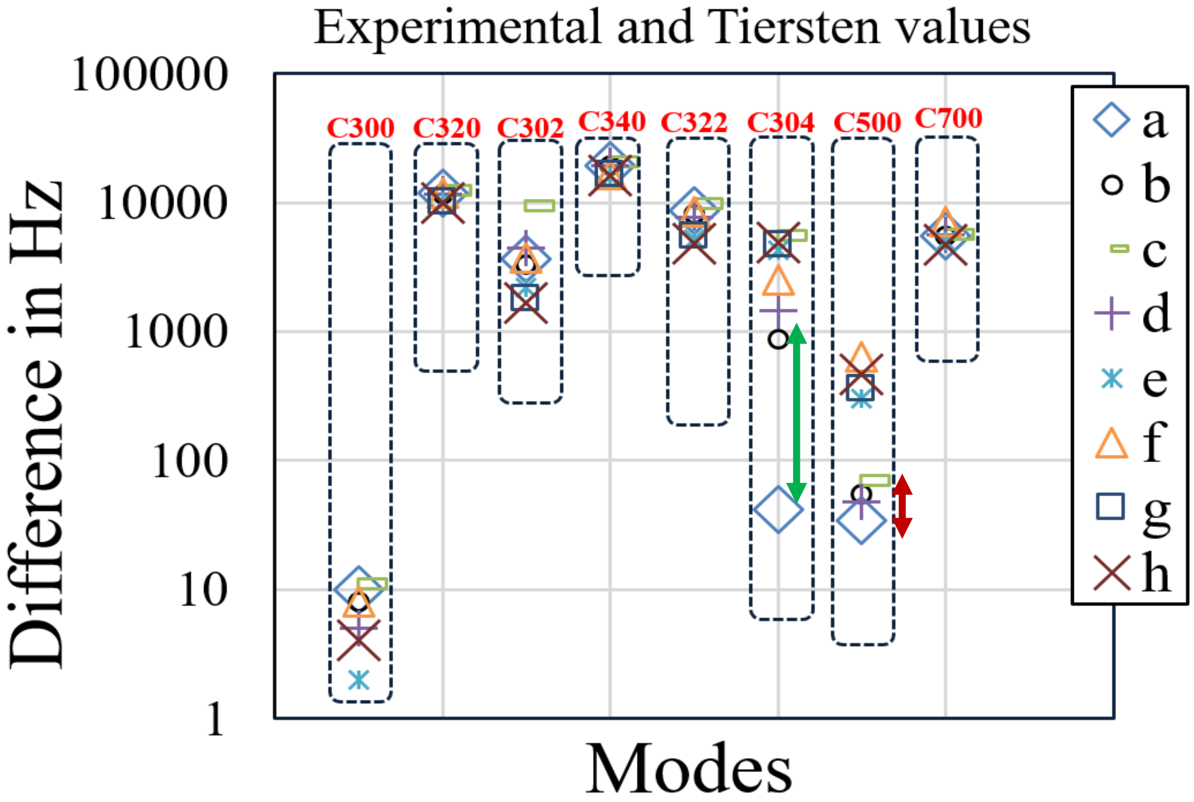
<u>Anharmonic modes of resonance</u>



Results:

- ➢ No significant relation seen between Q-factor and Phase noise- both in harmonic and anharmonic modes.

Measurement at 353 K



Difference in kHz

Difference in 10’s of Hz

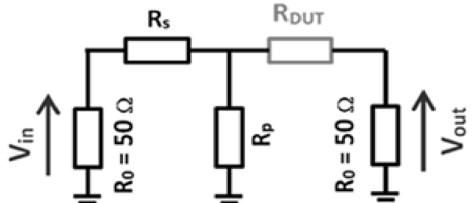
Results:

The existence of differences in resonant frequencies in anharmonic modes did not allow to do noise measurements.

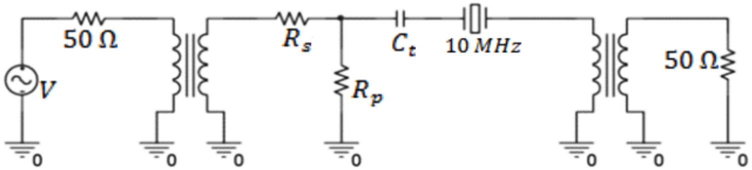
Frequency difference chart, exp. and Tiersten’s

Freqs.	Resos.	C300	C320	C302	C340	C322	C304	C500	C700
(Hz)	a	10	11743	3643	19477	8626	42	34	5491
353K	b	8	11709	3320	19360	8130	874	55	5469
	c	11	12183	9312	20564	9730	5550	70	5652
	d	5	11593	4411	19422	7752	1464	48	5601
	e	2	10119	2205	15890	5191	4262	300	5011
	f	8	11718	3750	16667	8546	2509	639	7124
	g	0	10203	1782	16598	5524	4683	359	0
	h	4	9974	1675	16177	4779	4871	468	4700

Alternative method for phase noise measurement technic on **LGT (Langatate) resonators***

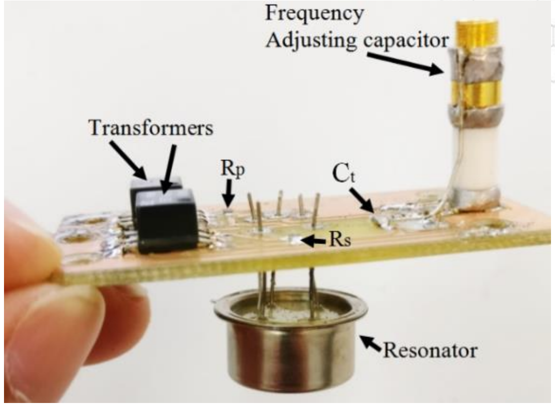


Classical impedance matching circuit

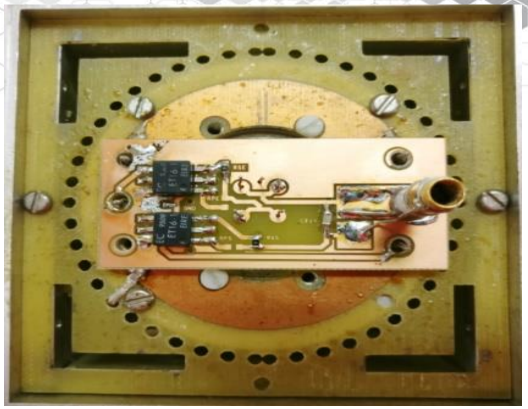


Modified impedance matching circuit

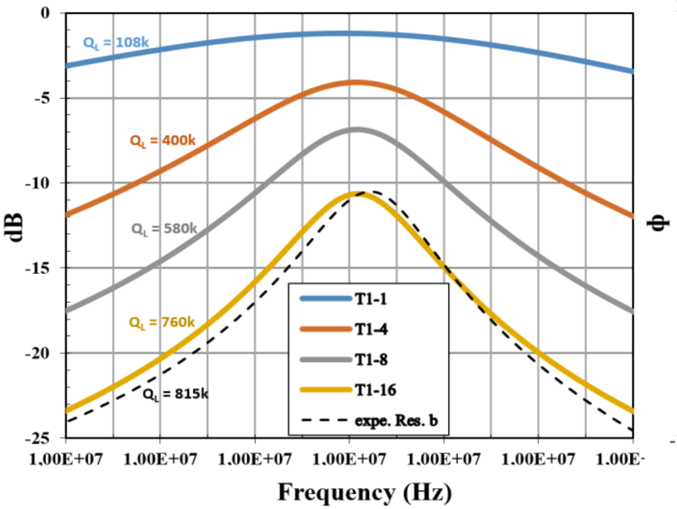
Because of low intrinsic impedance around 10 ohms

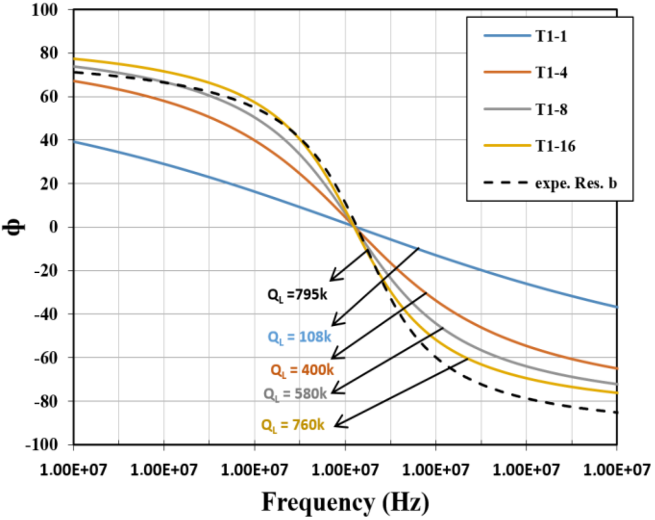


Resonator mounted in circuit



Resonator in oven

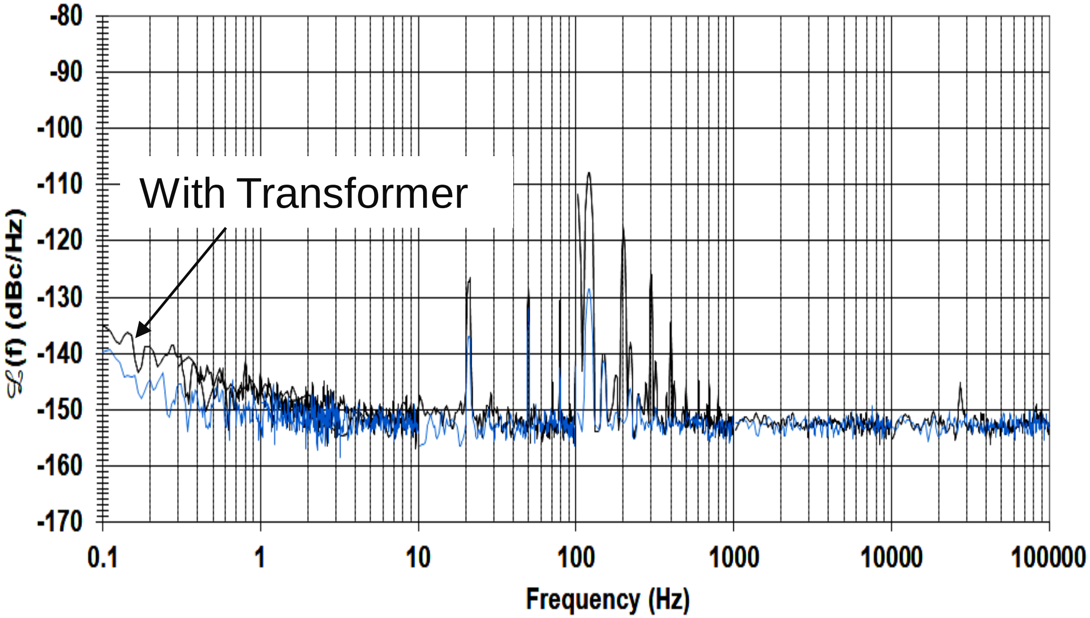




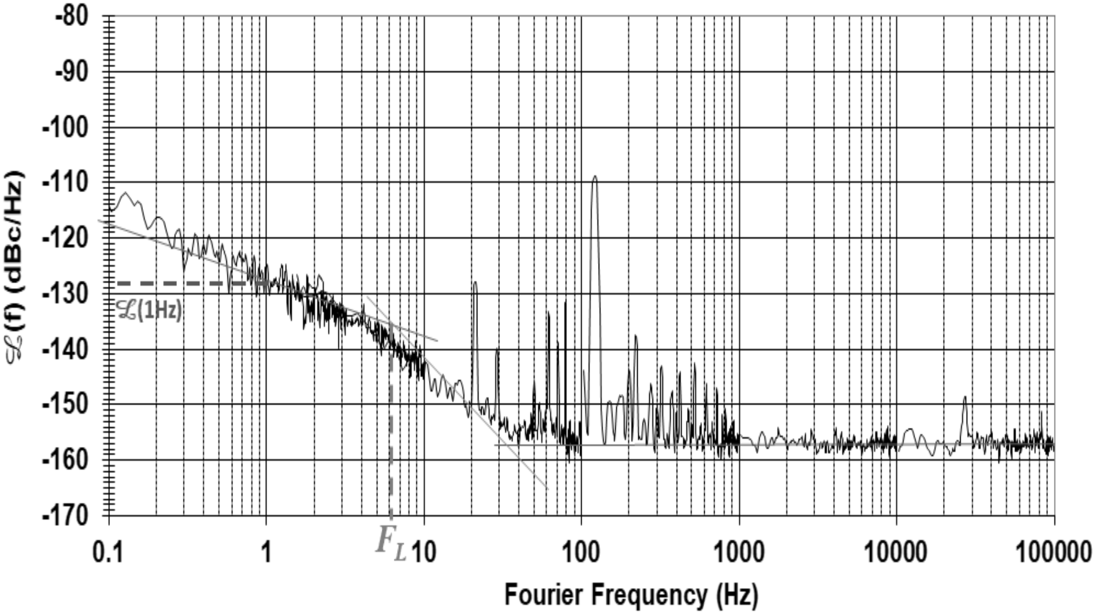
Results:

- Improved loaded Q factor.
- Almost 68% of the unloaded Q factor.
- Allowed Phase noise measurement.

*__A. Pokharel__, F. Sthal, E. Vaillant, J. Imbaud, J.J. Boy, F. X. Esnault, G. Cibiel, "Study of the phase noise of Langatate crystal resonators", Proc. European Frequency and Time Forum, Torino, Italy, 9-12 April, pp. 37-40, 2018.



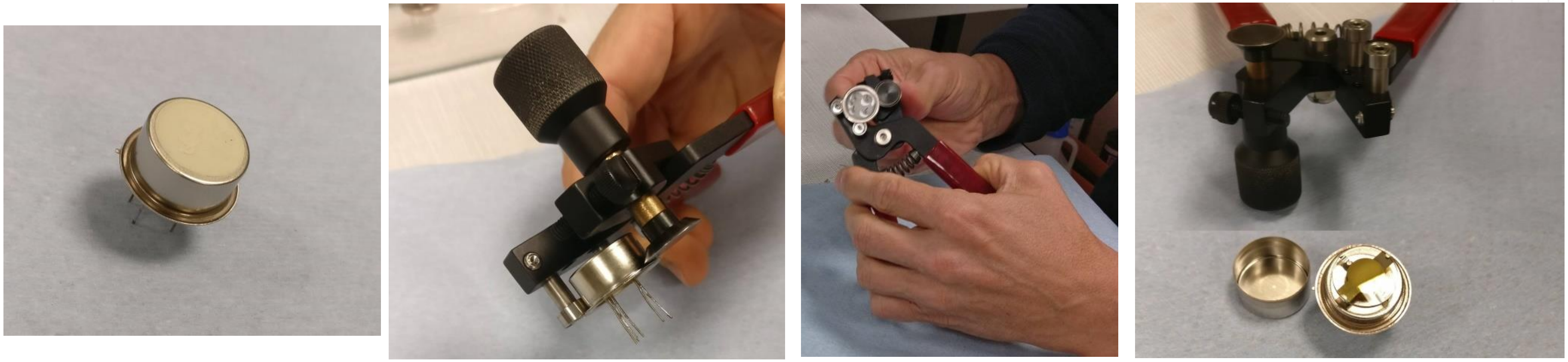
- No influence of the transfomers in the noise bench.
- Spurs seen after 100 Hz is due to frequency diff. Between synthesizer and reference.



- The value of shorterm stability is calculated for LGT around $3.04 \cdot 10^{-13}$ which is near to quartz crystal resonators which is around $5.2 \cdot 10^{-14}$.

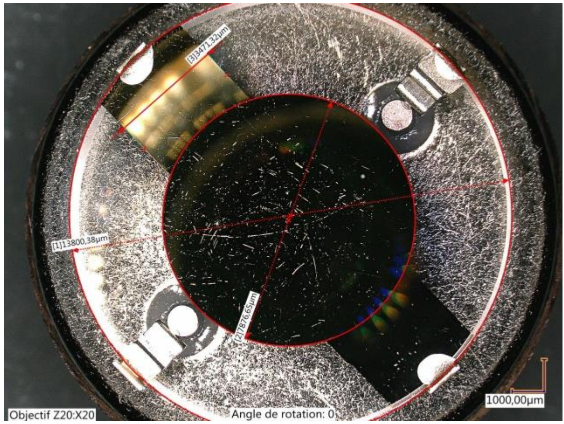
Analysis of the quartz crystal resonator by Reverse Engineering

Resonator Dismantling

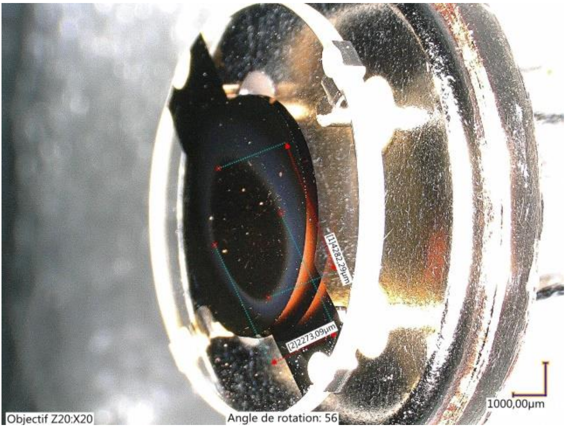


Optical analysis

Top View of the plate



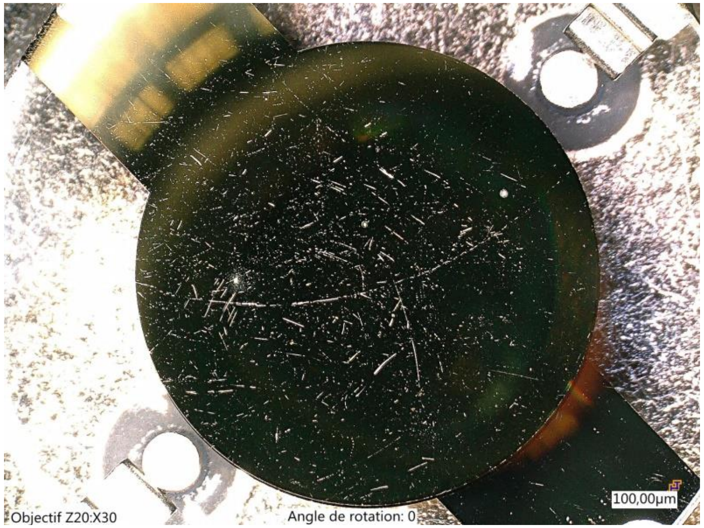
Side View of the plate



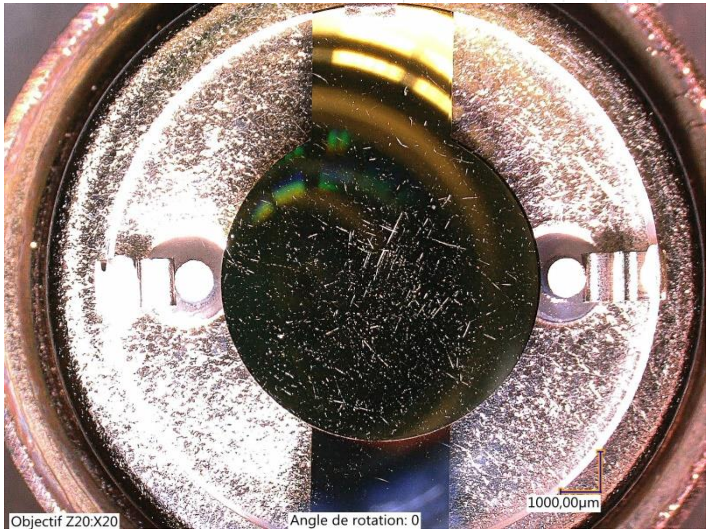
X-ray analysis remaining

Analysis of the quartz crystal resonator by Reverse Engineering

Scratches on electrode surface

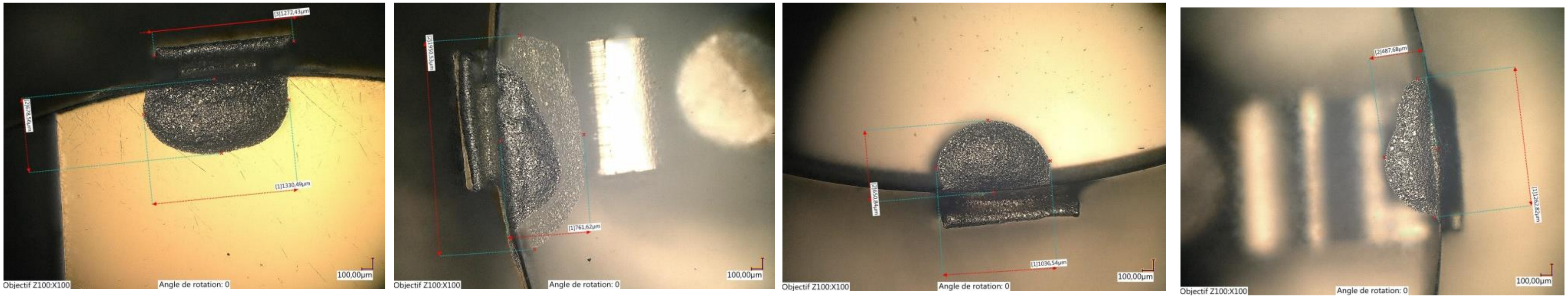


Bad in noise $1.8 \cdot 10^{-12}$



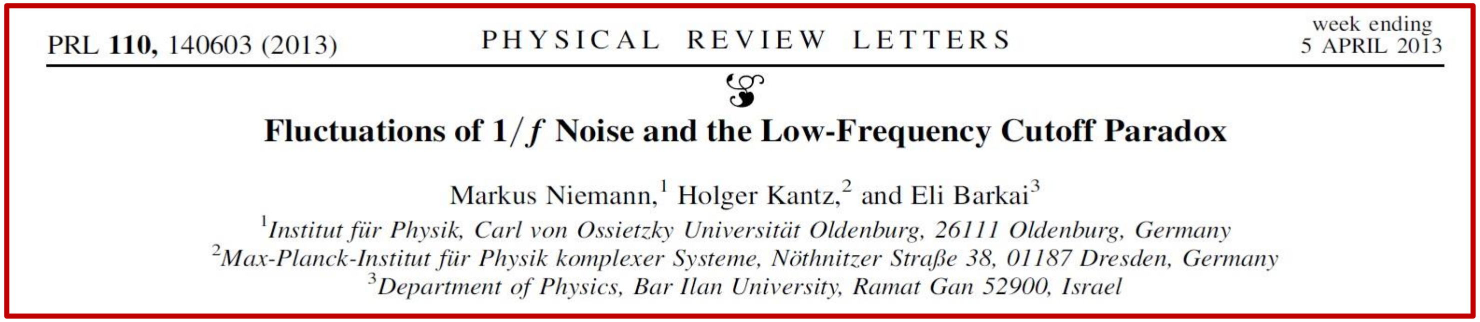
Good in noise $6.4 \cdot 10^{-14}$

Control of glue fixes



No significative difference

Modeling using Mittag-leffler distribution



PRL **110,** 140603 (2013) PHYSICAL REVIEW LETTERS week ending 5 APRIL 2013

Fluctuations of $1/f$ Noise and the Low-Frequency Cutoff Paradox

Markus Niemann,[1] Holger Kantz,[2] and Eli Barkai[3]

[1]Institut für Physik, Carl von Ossietzky Universität Oldenburg, 26111 Oldenburg, Germany
[2]Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden, Germany
[3]Department of Physics, Bar Ilan University, Ramat Gan 52900, Israel

- $1/f^{\delta}$ noise spectrum comes from power law waiting times in microstates:
  $\boldsymbol{\Psi(\tau)= \tau^{-(1+\alpha)}}$, $\mathbf{0<\alpha<1}$ **where** $\boldsymbol{\delta = 2-\alpha}$

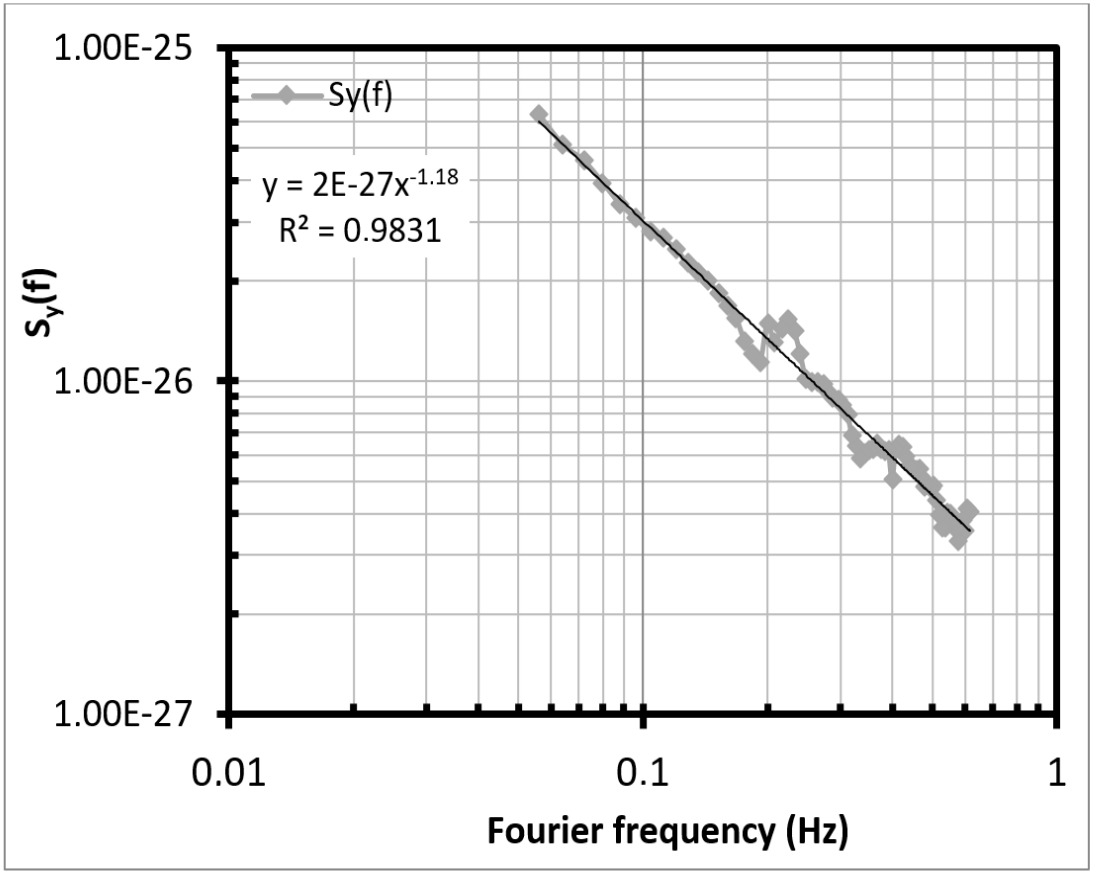
- For a given set of 48 measurements on the same pair of resonators, we compute $\xi_i = \frac{S_y(f_i)}{\langle S_y(f_i)\rangle}$, for frequencies f_i in an interval where $S_y(f) \propto {}^{1}/_{f^{\delta}}$ with $\delta \approx 1$.

- Niemann et al. $\Longrightarrow$ distribution of ξ_i should be exponential and $\mathrm{M} = \frac{1}{p}\sum_{i=1}^{p} \xi_i$ should be distributed according to a Mittag-Lefler distribution $Y_\alpha(M)$ with parameter α

$Y_\alpha(M) = \frac{\Gamma^{1/\alpha}(1+\alpha)}{\alpha M^{1+\frac{1}{\alpha}}}\, l_\alpha\left[\frac{\Gamma^{1/\alpha}(1+\alpha)}{M^{1/\alpha}}\right]$, l_α-Lévy dist. for $0<\alpha<1$, and $\mathrm{M} = \frac{1}{p}\sum_{i=1}^{p}\frac{S_y(f_i)}{\langle S_y(f_i)\rangle}$

Experimental results and observations

Log-log plot of PSD vs Fourier frequency for the average of the 48 measurements[1]



$$\langle S_y(f)\rangle \propto {}^{1}/_{f^{\delta}} \Longrightarrow \delta \approx 1.18$$

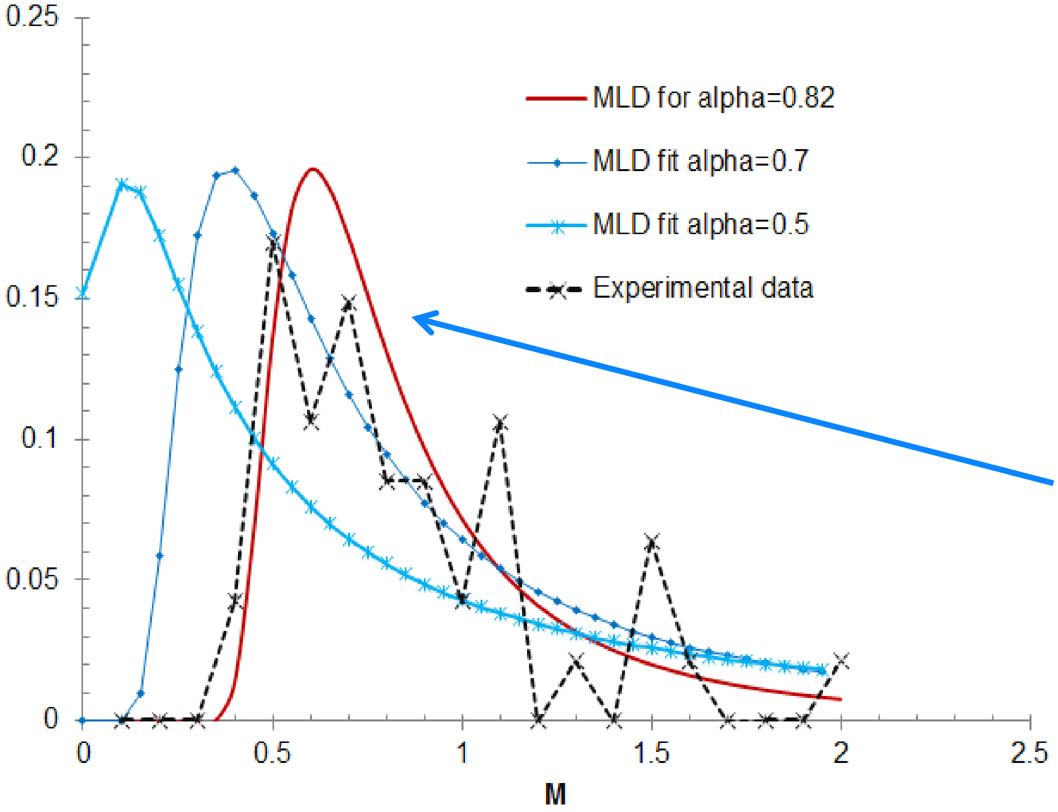
Niemann et al.:

$$\alpha = 2 - \delta \Longrightarrow \alpha = 0.82$$

[1] **A. Pokharel**, M. Devel, J. Imbaud, F. X. Esnault, G. Cibiel, F. Sthal, "Modeling of 1/f Phase noise on ultra-stable quartz crystal resonators using Mittag-Leffler distribution", Proc. Joint Meeting IEEE Int. Freq. Cont. Symp. and European Frequency and Time Forum, Orlando, FL, USA, 14-18 April, 2019, in press.

Experimental results and observations

Exp. data fit using Mittag-leffler distribution [1]



$$M = \frac{1}{p}\sum_{i=1}^{p}\frac{S_y(f_i)}{\langle S_y(f_i)\rangle}$$

Number of frequencies $p = 400$
$\langle\ \ \rangle$ taken on 48 samples

Fitting the experimental normalized data to Mittag-leffler distribution with **α**=0.82

Good agreement !

[1] **A. Pokharel**, M. Devel, J. Imbaud, F. X. Esnault, G. Cibiel, F. Sthal, "Modeling of 1/f Phase noise on ultra-stable quartz crystal resonators using Mittag-Leffler distribution", Proc. Joint Meeting IEEE Int. Freq. Cont. Symp. and European Frequency and Time Forum, Orlando, FL, USA, 14-18 April, 2019, in press.

Conclusion and perspectives

- We measured $1/f$ flicker noise in quartz crystal resonator pairs provided from CNES and compared the experimental resonant frequencies with those calculated from Tiersten's formula
- The noise measurement in anharmonic modes were not possible- difference in resonant frequencies of resonator pairs.
- We successfully implemented alternative method to measure phase noise in LGT resonators
- We dismantled a pair of resonator and did optical analysis- **X-ray analysis is ongoing.**
- Histogram of normalized power spectra seems to follow a PDF given by Mittag-Leffler distribution with an exponent similar to the one given by $1/f$ direct fit of the spectrum.
- Our results seem to support an approximate adequacy of power law intermittency mechanism to give an account of $1/f$ noise in quartz resonators.
- Needs to make more measurements on lastly measured resonator pair and other resonator pairs too like Good-average, Good-bad and Good-good.

Thank you.

ANR-10-LABX-48-01

ANR-11-EQPX-0033-OSC-IMP

ANR-17-EURE-0002