Determination of the fine-structure constant with 81 parts-per-trillion accuracy

Zhibin YAO, Léo Morel, Pierre Cladé and Saïda Guellati-Khélifa

Laboratoire Kastler Brossel — Physique quantique et applications

CNRS

Collège de France 1530

Sorbonne Université

ENS École Normale Supérieure

The fine structure constant α and ratio $\frac{h}{m}$

- **Fine Structure Constant**

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c}$$

- **From the hydrogen atom model:**

$$hcR_\infty = \frac{1}{2} m_e c^2 \alpha^2$$

- R_∞ with relative uncertainty $\mathbf{2\times10^{-12}}$

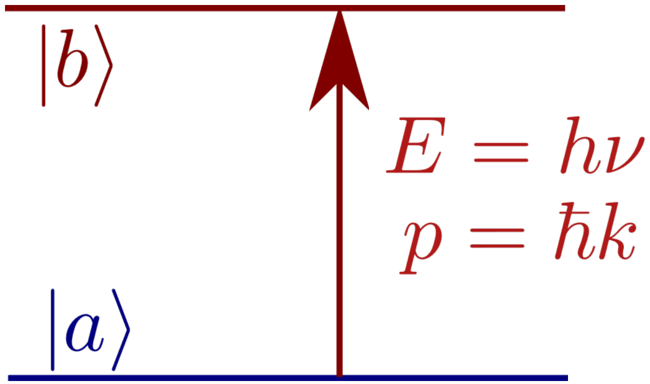
- $\frac{m_{Rb}}{m_e}$ with relative uncertainty 7.5×10^{-11}

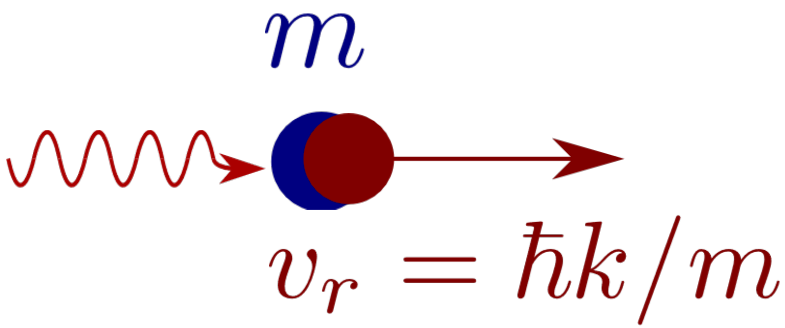
$$\Rightarrow \alpha^2 = \frac{2R_\infty}{c}\frac{m_{\mathrm{Rb}}}{m_e}\frac{h}{m_{\mathrm{Rb}}}$$

- $\frac{h}{m_{\mathrm{Rb}}}$ with relative uncertainty **? ? ?**

Recoil Velocity

Laboratoire Kastler Brossel
Physique quantique et applications

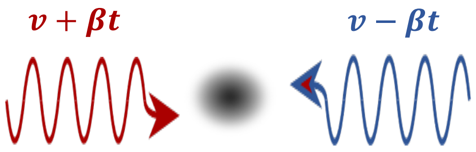




We are using the atom interferometer to measure this recoil velocity.

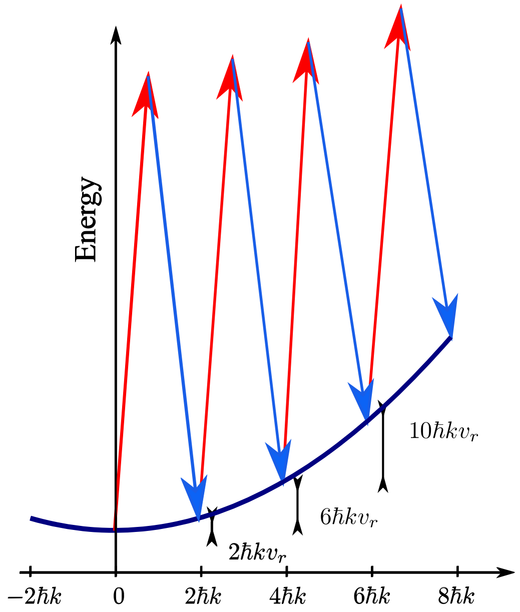
Accurate determination of the fine structure constant for testing the Standard model

Coherent Acceleration : Bloch oscillation



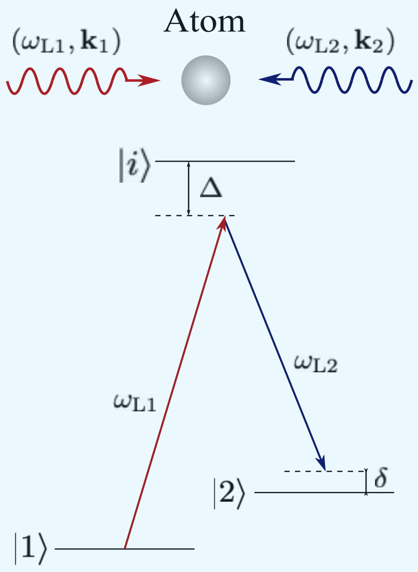
$$p_0 \rightarrow p_0 + 2\hbar k$$
$$p_0 \rightarrow p_0 + 2N_{bloch}\hbar k$$

1000 photon momemtum (6 m/s) in 6 ms



- High momentum transfer efficiency: 99.95% per recoil
- Precise control of the velocity and the position of the atoms

- Stimulated Raman transition

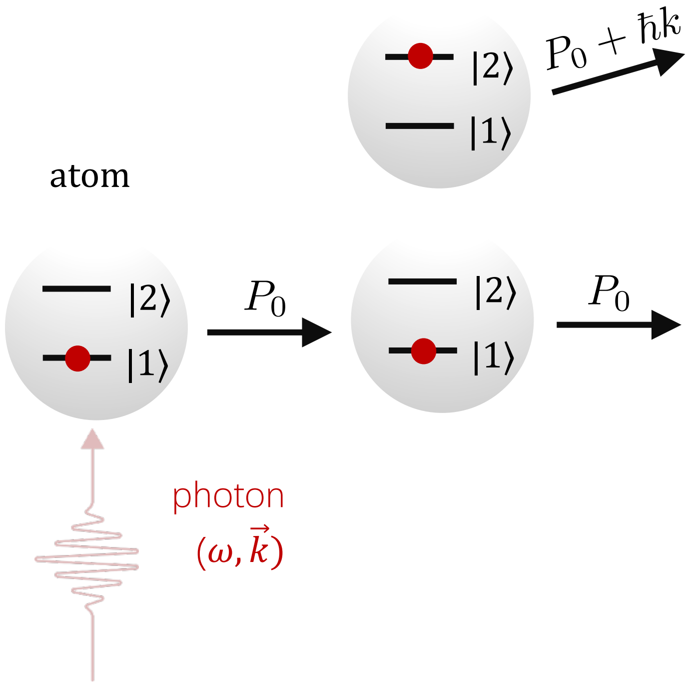


$\Delta \gg \delta$ and $\Delta \gg \Gamma$

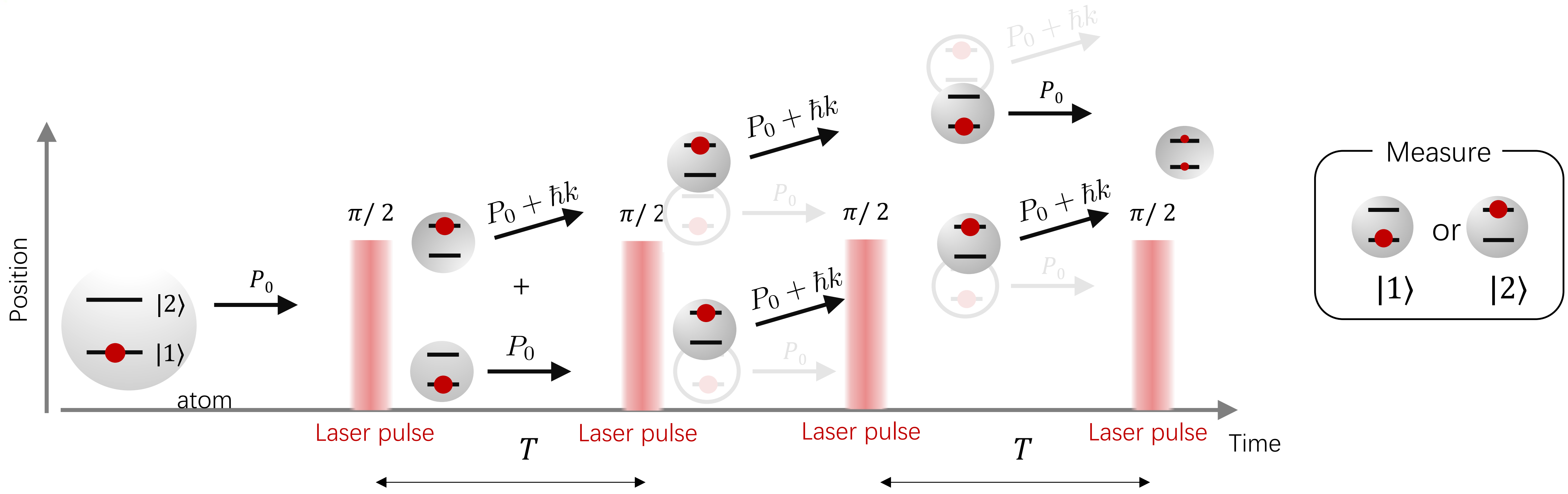
$$\Omega\tau = \frac{\pi}{2} \longrightarrow \Psi(\tau) = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{-i\omega_1\tau}\,|1, \mathbf{p}_0\rangle + \mathrm{e}^{i\Delta\phi_L}\mathrm{e}^{-i\omega_2\tau}\mathrm{e}^{-i\pi/2}\,|2, \mathbf{p}_0 + \hbar\mathbf{k}\rangle\right)$$

- Contra-propagation laser beams: velocity sensitive Raman transitions

- **Atomic beam splitter**



Ramsey-Bordé atom Interferometer

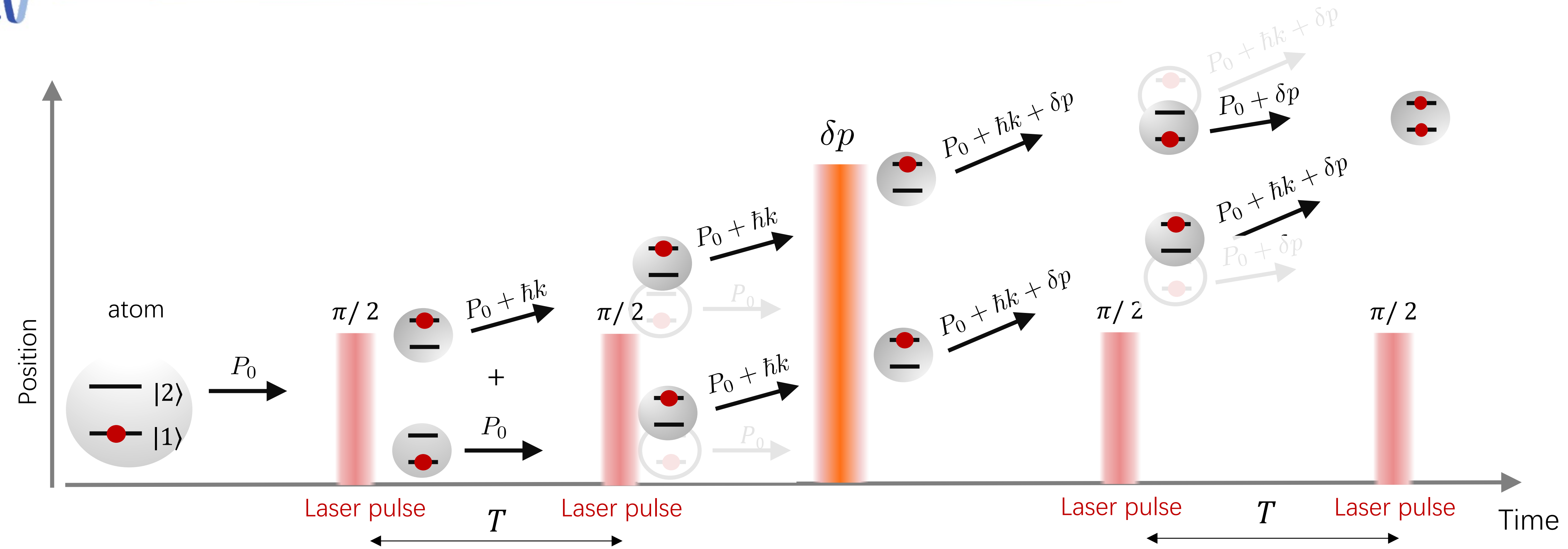


Probability to find atoms in $|2\rangle$

$$P_2 = \frac{1 + \cos\left(\Delta\Phi_{\mathrm{at}} + \Delta\Phi_{\mathrm{Las}}\right)}{2}$$

Free propagation: $\mathrm{e}^{-i\frac{E_{1,2}}{\hbar}t}$, $E_{1,2}$ = internal energy + kinetic energy

Quantum velocity sensor based on atom interferometry



Atomic phase in the **upper** branch

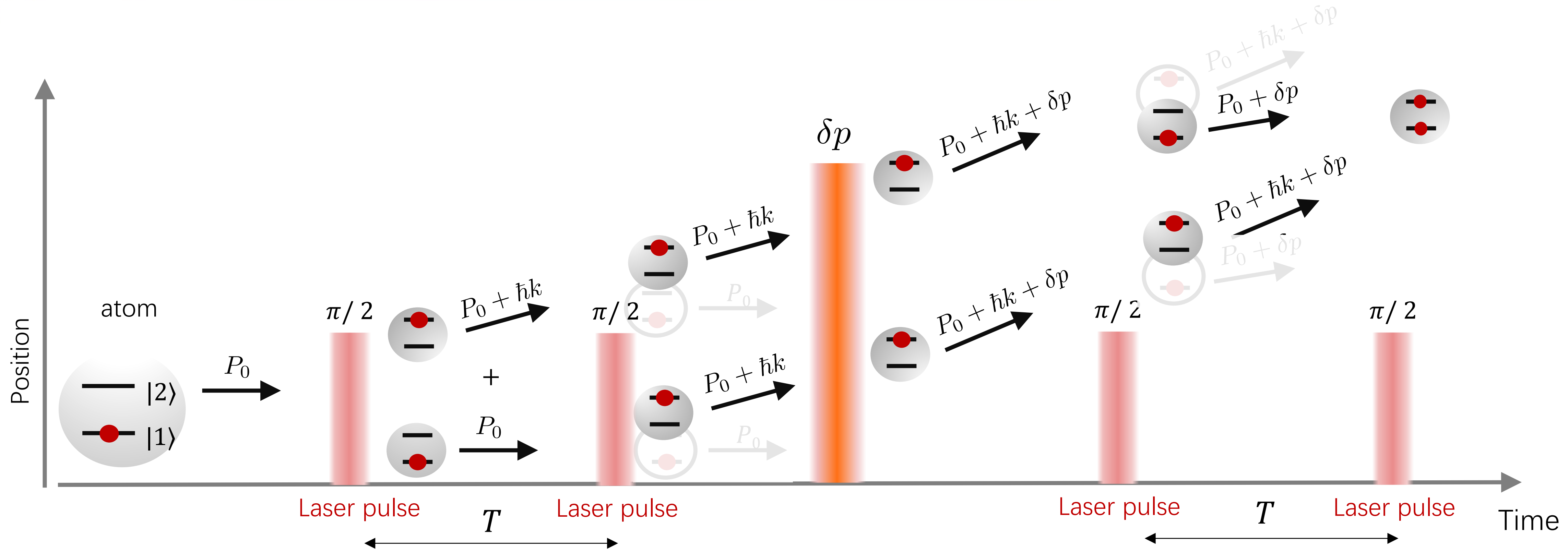
$$\Phi_{up} = \left(\omega_2 + \frac{1}{\hbar}\frac{(P_0 + \hbar k)^2}{2m}\right) T + \left(\omega_1 + \frac{1}{\hbar}\frac{(P_0 + \delta p)^2}{2m}\right) T$$

Atomic phase in the **lower** branch

$$\Phi_L = \left(\omega_1 + \frac{1}{\hbar}\frac{P_0^{\,2}}{2m}\right) T + \left(\omega_2 + \frac{1}{\hbar}\frac{(P_0 + \hbar k + \delta p)^2}{2m}\right) T$$

$$\Delta\Phi_{at} = \Phi_{up} - \Phi_L = T \times k \times \delta v, \text{ where } \delta v = N v_r \longrightarrow \text{ measure of the recoil velociy.}$$

Quantum velocity sensor based on atom interferometry



$$\Delta\Phi_{\rm at} = T \times k \times \delta v = \frac{\Delta z \times m\delta v}{\hbar}$$

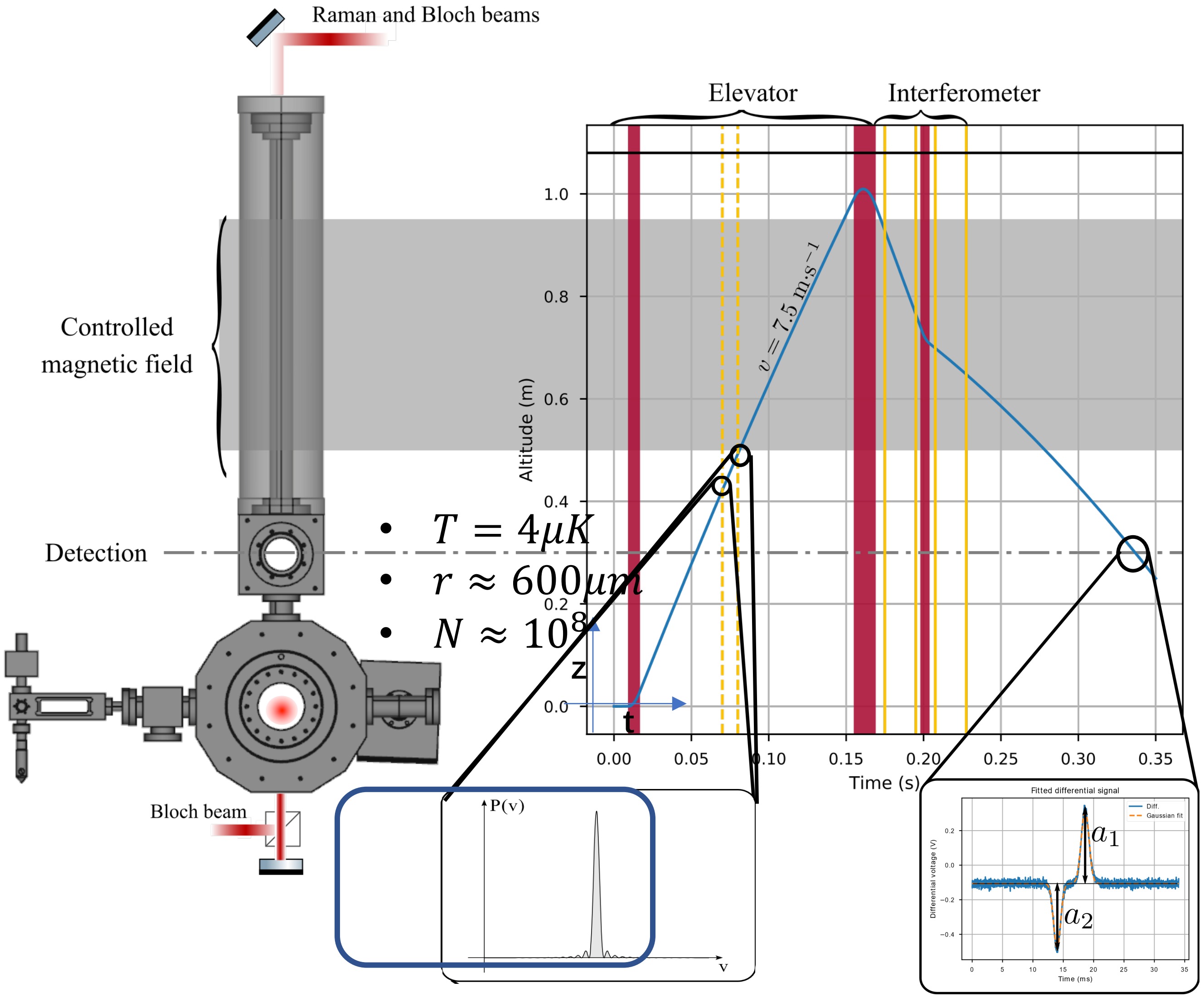
Sensitivity: $\delta z = 250\ \mu m \longrightarrow 3\ \mu m \cdot s^{-1} \cdot rad^{-1}$

$$\Delta\Phi = T\,(k\delta v - 2\pi\delta f_{\rm R})$$
$$\delta f_{\rm R} = f_{\rm R,2} - f_{\rm R,1}$$

Doppler shift

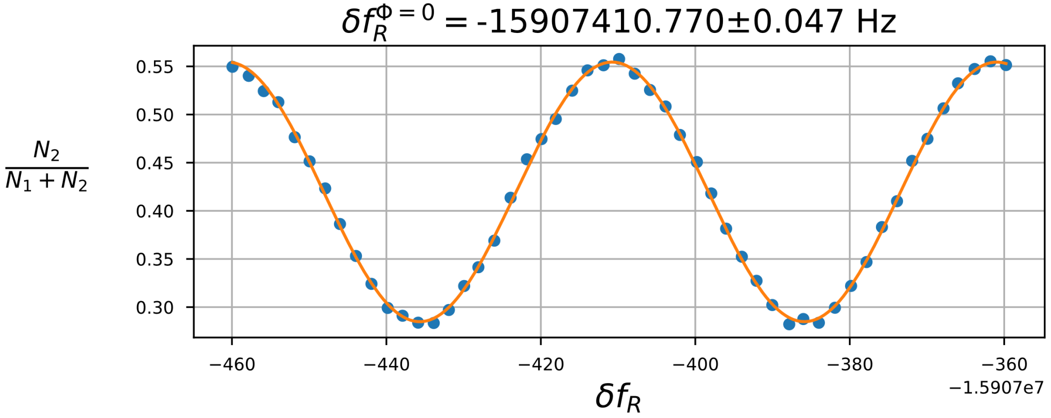
Experiment

Laboratoire Kastler Brossel
Physique quantique et applications



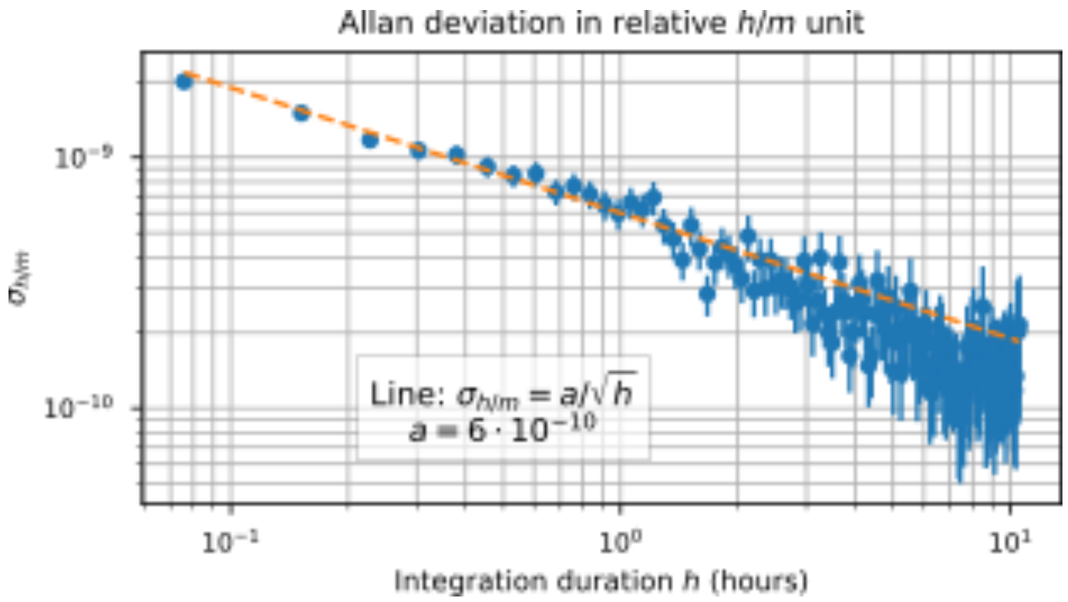
- $T = 4\mu K$
- $r \approx 600\mu m$
- $N \approx 10^8$

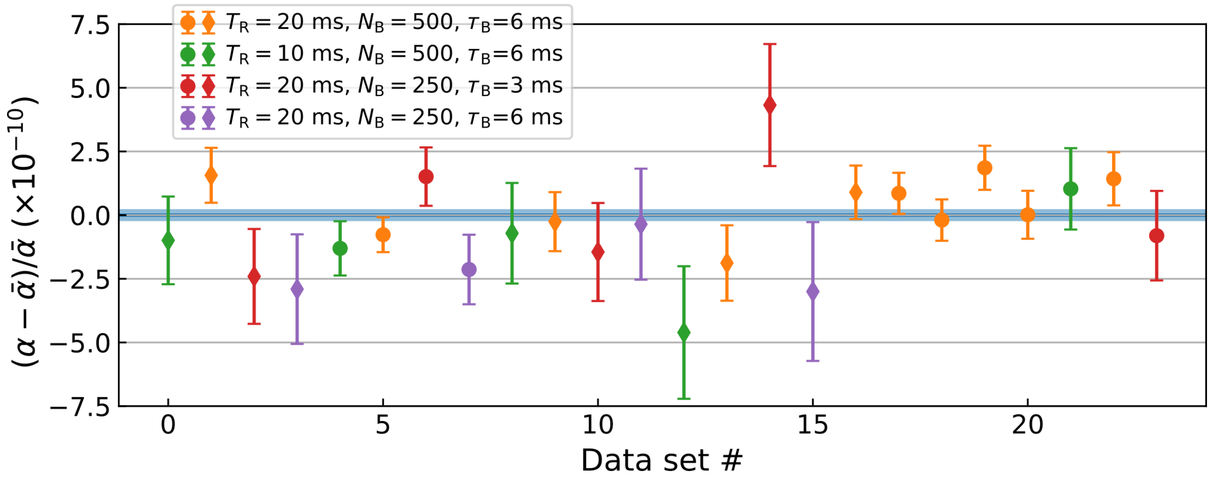
$T_R = 20\mathrm{ms}, \quad N_{recoil} = 1000$



$0.047Hz \sim 20nm/s \rightarrow 3 \cdot 10^{-9} on\ \frac{h}{m}$

Results


Allan deviation in relative h/m unit
$\sigma_{h/m}$
10^{-9}
10^{-10}
Line: $\sigma_{h/m} = a/\sqrt{h}$
$a = 6 \cdot 10^{-10}$
10^{-1}
10^{0}
10^{1}
Integration duration h (hours)


$T_R = 20$ ms, $N_B = 500$, τ_B=6 ms
$T_R = 10$ ms, $N_B = 500$, τ_B=6 ms
$T_R = 20$ ms, $N_B = 250$, τ_B=3 ms
$T_R = 20$ ms, $N_B = 250$, τ_B=6 ms
$(\alpha - \bar{\alpha})/\bar{\alpha}$ ($\times 10^{-10}$)
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0
5
10
15
20
Data set #

Error buget

Laboratoire Kastler Brossel
Physique quantique et applications

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of ^{87}Rb [16] : $86.909\,180\,531\,0(60)$		3.5
Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$		1.5
Rydberg constant [14] : $10\,973\,731.568\,160(21)\text{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035\,999\,206(11)$		8.1

L. Morel, Z. Yao, P. Cladé, S.Guellati-Khelifa
Nature 588 (7836), 61-65 (2020)

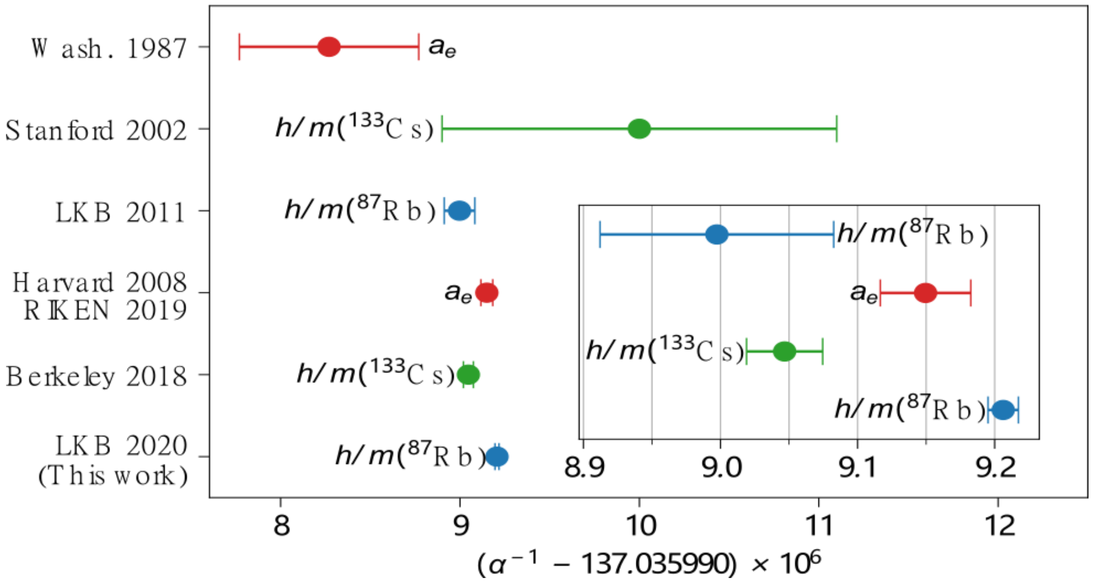
Quantum Electro-dynamics Standard Model

Laboratoire Kastler Brossel
Physique quantique et applications

Current status

$$\delta a_e = a_{e,\exp} - a_e(\alpha_{\mathrm{Berkeley}}) = -0.88(0.36) \times 10^{-12} (2.5\sigma)$$

$$\delta a_e = a_{e,\exp} - a_e(\alpha_{\mathrm{LKB2020}}) = 0.48(0.30) \times 10^{-12} (1.6\sigma)$$



Electron magnetic moment anomaly

$$a_e = \frac{g_e - 2}{2}$$

$$a_{e,exp} = 0.00115965218073(28)$$

Standard model prediction

$$a_e = a_e(\mathrm{QED}) + a_e(\mathrm{Had}) + a_e(\mathrm{Weak})$$

$$a_e(\mathrm{QED}) = \sum_{n=1}^{\infty} A^{2n} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{2n}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

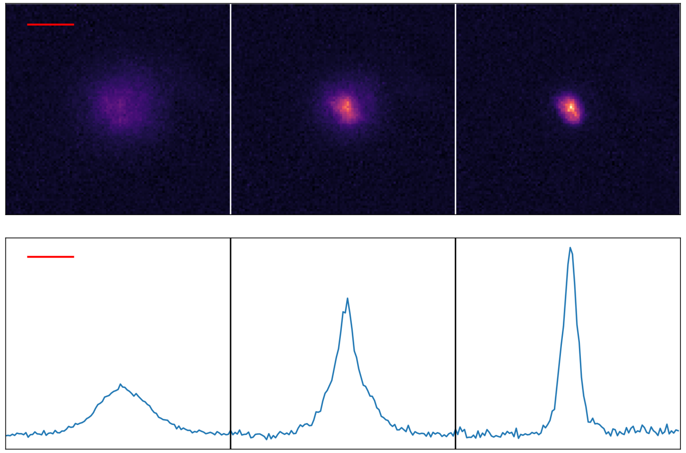
Laboratoire Kastler Brossel
Physique quantique et applications

Conclusions

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of ^{87}Rb [16] : $86.909\,180\,531\,0(60)$		3.5
Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$		1.5
Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035\,999\,206(11)$		8.1

- New determination of the fine-structure constant with a relative uncertainty of 8.1×10^{-11}
- Three new systematic effects
- The large discrepancy ($5.4\ \sigma$) with the caesium recoil measurement needs to be clarified

Prospects



- BEC
- Mean-Field

Thank you !