Metrology and statistics: from clocks to millisecond pulsars

Thesis supervisor: Prof. François Vernotte, FEMTO-ST/Time and Frequency Department, Observatory THETA (Besançon, France), francois.vernotte@obs-besancon.fr

Candidate profile: Master of Physics or Applied Mathematics

Context

Time is the physical quantity which is measured with the greatest precision. Recent advances in atomic clocks have made it possible to reach relative stabilities of the order of a few 10^{-18}, corresponding to an uncertainty of one second over the age of the Universe. However, just because these uncertainties become ridiculously small does not mean that they have to be neglected, on the contrary. They even pose difficult statistical problems and brought clockmakers, engineers and then researchers to invent metrology, the science of measurement. Indeed, it is important to distinguish concepts as stability, the ability to deliver constant units of time, and accuracy, the ability to deliver a unit of time according to its definition. The characterization of a clock thus involves the creation and use of statistical estimators adapted to identify and measure all these concepts, which is all the more delicate as the accuracy is high [1]. In Besançon, time-frequency research, heir to the city's clockmaking past, is internationally recognized as being at the world state of the art world, whether to manufacture new clocks [2], such as trapped ion clocks, for example, or to measure them [3]. Today, this problem is all the more important because the definition of the second is likely to change in the next few years to rely on the physical principles at work in modern clocks. On the other hand, the control of time is a fundamental issue in the space field. All measurements of position and velocity of satellites or interplanetary probes are based on ultra-precise measurements of time and frequency [4]. Similarly, positioning and navigation by satellite (GPS, Galileo) rely on the measurement of the transit time of the signals emitted by these satellites. Since these signals travel at the speed of light, an error of 3 billionths of a second is reflected in a positioning error of one meter [5]. Finally, this project is also motivated by the study of periodic astronomical phenomena, in particular the reception of pulses originating from millisecond pulsars, whose very fine irregularities expected by some theorists could, if they were detected in the flow of measurements obtained, have amazing fallout in both theoretical physics and cosmology [6]. A new field for metrology of time is opening up, demanding new statistical tools capable of detecting the most minute disturbances. The recognized experience in this field within FEMTO-ST and Besançon Observatory associated with the statisticians of the Laboratory of Mathematics of Besançon will certainly be able to meet this challenge. It will also be an opportunity to train a young researcher in the context of a PhD in order to transmit and enrich the knowledge accumulated in this field at the confluence of clockmaking, physics, space domain and astronomy.

Work program

The first part of the expected works concerns mainly the fine characterization and the improvement of the methods of estimation of the stability of a set of clocks. These methods, recently developed in the time-frequency department of FEMTO-ST and in Observatory THETA, are revolutionizing the metrology of time and frequencies. These methods aim, on the one hand, to reject most of the measurement noise in order to better highlight the instabilities of the clocks and, on the other hand, to compare a set of at least 3 clocks in order to deduce the stability of each of the clocks. They are based on two statistical properties: the use of least squares which allows an optimal rejection of white noise with the development of the "parabolic variance" [7] as well as the use of the signal covariances which allows to keep only the common part of the signals ("cross-spectra" on the one hand [8], "covariance of Groslambert" [1] on the other hand). These tools have already been partially characterized in terms of signal processing but a detailed analysis in terms of Bayesian statistics remains to be done, in particular to obtain reliable confidence intervals around the measurements made by these estimators. On the other hand, it will also be very useful to combine these tools to obtain more efficient estimators. One can for example imagine the "parabolic covariance" which would combine optimal rejection of the white noise as well as the non-common part (decorrelated) of the two signals. To carry out these tasks, the doctoral student should have a thorough knowledge of the basics of signal processing (Fourier transformation, convolutions, impulse responses, transfer functions, Z transforms, etc.) as well as of the measurement statistical analysis (estimation, statistical tests, inverse problem, Bayesian analysis, etc.).

The second part concerns the application of these new tools on three main fields, already identified, others that may occur during the thesis.
First of all, we need to be able to estimate the stability of the new clocks in development in the FEMTO-ST TF department (trapped ion clocks, superradiant laser) which are potentially destined to break world stability records. The problem then is to estimate the stability of a clock that is better than all the others! Two possibilities exist: to build at least two (or even three) identical clocks and to compare them, which is practically unrealistic, or to study the clock simultaneously with two independent instruments connected to two equally independent clocks having a stability slightly lower than that of the clock under test. Then, the use of optimal estimators will allow to obtain the stability of this new clock. Such work will naturally be part of the activities of the FEMTO-ST Time-Frequency Department and the PhD student will benefit from an ideal working environment.
These new estimators can also be used for the long-term monitoring of ultra-stable oscillators present in all satellites as well as the atomic clocks present in the satellites of the Galileo constellation. Here again, the idea is based on the simultaneous reception of the signals coming from the same satellite by two ground stations separated by several hundred km. The geographic distance will cause a decorrelation of the atmospheric effects disturbing the reception of the signals in the two stations and will allow to go up to the intrinsic stability of the clock of the satellite. This study will be conducted in collaboration with the colleagues of Geoazur (Observatory of the Côte d'Azur).
Finally, these new statistical tools could prove to be determining for another project in which we are involved: the analysis of millisecond pulsar timing. Here again, the idea rely on the simultaneous use of two distant radio-telescopes, for example Nançay in France and Jodrell Bank in England, to observe the same millisecond pulsar. In this context, the PhD student will work with the team at the Nançay astronomical station and in particular the post-doctoral fellow that we have just recruited jointly between FEMTO-ST and the Nançay station (Labex FIRST-TF funding).

As an extension of his research activity, the doctoral student could also participate in the development of SigmaTheta, the software suite for time-frequency metrology initiated at OSU THETA (https://theta.obs-besancon.fr/spip.php?article103, CeCILL license).

Progress of the thesis

- *1st year:* acquisition of the time and frequency concepts and tools (basic definitions, spectral analysis, Allan variance, etc.), of the methodology developed in the laboratory (back and forth between theoretical analysis and massive Monte-Carlo simulations), of the core strengths developed in Besançon (parabolic variance, cross-spectra, Groslambert covariance) and then of the current research (calculations of confidence intervals of specific estimators)
- *2nd year*: application to practical problems on FEMTO-ST atomic clocks, characterization of oscillators on board satellites and analysis of millisecond pulsar timing
- *3rd year*: deepening and mobility in some other laboratories with which we collaborate: for example, at the national level, Paris Observatory, CNES in Toulouse or Nançay radio astronomy station; at the European level, INRIM in Turin, PTB in Braunschweig, LTF in Neuchâtel or ESA in Noordwijk; at the international level, NIST in Boulder.

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